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Below—Row of bold-dialed Micromax Recorders, designed to show at a glance malt-roasting temperatures in Manitowoc, Wis., plant of Rahr Malting Co.

Above—General view of Rahr malt-roasting department. L&N thermocouple, installed in a roaster (see arrow), detects temperature and reports it to a Micromax Recorder.

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Vol. 241 APRIL, 1946 No. 4

PRELIMINARY CALCULATION OF THE MASS RATIO, RANGE, AND AVERAGE COEFFICIENTS OF EFFICIENCY OF A LONG-DISTANCE STRATOSPHERIC JET-PLANE.

BY

M. ZBIGNIEW KRZYWOBLOCKI, Dr.Ae.E., M.Ae.E., M.Ap.Math., Dipl.Ing., A.F.R.Ae.S., A.F.I.A.S.

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SUMMARY.

The mechanics of flight of a stratospheric jet-plane is in the infancy of its development, and very little has been done in this field to date. The problem is a very difficult one, mainly from the standpoint of mathematics. A large number of variable parameters appear in the differential equations of motion making their solution impossible. The mass of a jet-plane is variable, and the discrepancies are so great that they cannot be ignored. The height and the pressure and density of the air are also variable, and these variations cannot be neglected, especially when one considers heights of the order of 50 to 100 thousand feet. In the case of a jet-plane taking air from the outside, the air-fuel ratio by weight is also variable because of the variation in the oxygen percentage of the air with height. Finally, the changes in weight of the jet-plane cause the angle of attack to vary between rather wide limits.

In the present paper three periods of flight are taken into consideration with various simplifying assumptions: straight line climbing, horizontal flight, and straight line gliding. In each period, flight with subsonic and supersonic velocities is discussed. Equations for the mass ratio at the beginning and end of each period and for the distance covered in flight are derived. Even though several far-reaching simplifications were made, the calculations lead to hyperelliptic integrals of higher order. Some of them are solved; some are left for graphical integration in any particular case. Because of these simplifications the calculation may be considered as only a preliminary one. The equations for the average coefficients of efficiency in each period of flight are given next. The question of whether or not the plane has a crew does not enter this consideration. Some final remarks close the paper.

SYMBOLS.

F, reactional force of the exhaust gases.
m_f, mass of fuel used in 1 sec.
c, velocity of the exhaust gases with respect to the walls of the nozzle.
α, air fuel ratio by mass or by weight.
v_0, v_1, v, velocity of flight in the different periods.
k, coefficient.
W, instantaneous weight of the airplane.
g, acceleration of gravity at the height.
t, time.
c_L, c_D, lift and drag coefficients.
ϵ, ratio: (D/L).
ϕ_n, angle.
R, radius of the earth (assumed to be a sphere).
g_0, acceleration of gravity at sea level.

h, height.
W_0, W_1, W_2 . . . W_n, weight of plane at the beginning and end of the different periods of flight.
L, D, lift, drag.
ρ_0, ρ, density of the air at seal level and at a certain height.
S, wing area.
s, distance of flight along the flight path.
k_1, k_2 . . . k_n, constants.
v_{i0}, v_{i1}, v_{in}, initial velocities in each period.
l_n, horizontal distance covered in each period.
c_{LS}, c_{DS}, lift and drag coefficients in the supersonic range.
A, B . . ., $\bar{A}$, $\bar{B}$ . . ., coefficients.
F_c, centrifugal force.
K_n, kinetic energy at the end of the period.
E_n, potential energy at the end of the period.
M_n, energy due to the work in the period.
Q_n, energy contents of the burned fuel.
L_n, energy lost due to decrease in weight.
Q, number of kilocalories per pound of fuel.
c_t, theoretical velocity of the exhaust gases.
η_n, average coefficient of efficiency.

INTRODUCTION.

The mechanics of flight of a strotospheric jet-plane is in its infancy, and very little has been done in this field to date. Some cases of rocket-plane flight under many simplifying assumptions may be found in the known book of Sänger, "Raketenflugtechnik." He was probably the first one to attack the problem of flight of a stratospheric rocket-plane at very great heights and with very high velocities; but very few other papers on this subject can be found in the available literature. The heights involved in these cases are of the order of 50,000 to 100,000 feet; the velocities of flight are much higher than the velocities of sound. The problem is very difficult from the standpoint of mathematics, for the differential equations of motion contain a large quantity of variable parameters making the solution of these equations impossible. In the first place, the variations in the mass of the plane and the mass ratio at the beginning and end of the period of flight considered are great. The height of flight, and the density and pressure of the air also vary, and these last parameters change so much in the high altitudes which are taken into account that they cannot be neglected. Actually the law governing the variation of density and pressure at high altitudes is not known precisely, so that several approximate formulae must be used. In the case of a jet-plane taking air from the outside, the air-fuel ratio by weight also varies because of the variation of the per-

centage of oxygen in the air with height. Finally the angle of attack and the drag and lift coefficients vary because of changes in the weight of the plane.

It is obvious from the above that taking into account all the variable parameters leads to differential equations which cannot be solved. For this reason the solutions obtained may be considered only as preliminary, since various simplifying assumptions had to be introduced in order to enable solution of the equations.

In the present paper three periods of flight are taken into consideration: straight line climbing, horizontal flight and straight line gliding. In each period flight with subsonic and supersonic velocities is considered. Two kinds of plane are taken into account: the rocket-plane in which oxygen necessary to the combustion process is contained inside the plane in a liquid state, and the jet-plane taking air from the outside. The simplest possible type of jet-motor is considered, i.e., a nozzle with an air intake in the front and with a fuel injection device. Air enters in the front with the velocity of flight and leaves with the velocity of the exhaust gases by virtue of the combustion of the fuel. For each period of flight and for the two above mentioned types of planes, equations are derived for the mass ratio at the beginning and end of the period and for the distance of flight covered. Some far-reaching simplifications were made: the mass of the plane is assumed to be constant in each period and equal to the mean of the values at the beginning and end of the period. Simple formulae were applied for the variation of density with height and for the change in lift coefficient with supersonic velocity, where the value of this coefficient is not constant but depends on the velocity of flight. In subsonic flight the value c_L is assumed to be constant. The fineness ratio $c_L:c_D$ and the air fuel ratio by weight are assumed to be constant in all cases of flight independent of the height and velocity. The pressure of the exhaust gases at the mouth of the nozzle was assumed to be equal to the pressure of the outside air. However, even with these simplifications the calculations lead to hyperelliptic integrals sometimes of very high order. To prevent obscuring the paper by long mathematical computations, some of these integrals are left unsolved. They may be solved in any particular case by precise methods or by graphical integration.

Next the optimum height of transition from subsonic to supersonic velocity was calculated in an algebraic way and fair agreement with the result obtained by Sänger was obtained. Sänger solved this problem in a graphical way. The average coefficient of efficiency for each period was calculated under the assumption that this coefficient represents the ratio of all the energies (potential, kinetic and mechanical due to the work done) at the end of the period to the energy content of the fuel burned in this period plus the lost energy of the plane. Finally the value of the average coefficient of efficiency for the whole propelled

period was calculated. Some remarks concerning the possibility of a graphical representation of the results close the paper.

1. *Climbing of a Jet-Plane.*

(a) *Subsonic velocity.*

The reaction force of the exhaust gases is:

$$F = m_f c + \alpha m_f (c - v). \tag{1}$$

Assume that:

$$m_f = k \frac{W}{g} \tag{2}$$

where the coefficient k is not assumed to be constant. Thus one may write:

$$dW = -kW\,dt, \qquad \log W = -\int_0^t k\,dt + \text{const.} \tag{3}$$

With the initial conditions that $t = 0$, $W = W_0$, one obtains:

$$\frac{W}{W_0} = e^{-\int_0^t k\,dt}. \tag{4}$$

In all the calculations, assume $\alpha = \text{const.}$, $\epsilon = \text{const.}$, and for the subsonic velocity $c_D = \text{const.}$, $c_L = \text{const.}$ Assume also straight line climbing at the angle ϕ_1 to the level, and take g as a constant value in the whole climbing period although g actually varies according to the formula $g : g_0 = R^2 : (R + h)^2$. The weight of the plane at the beginning of this flight period is W_0 and at the end, W_1. Thus one obtains:

$$F = W \sin \phi_1 + D + \frac{W}{g}\frac{dv}{dt}, \tag{5}$$

$$L = W \cos \phi_1, \qquad D = \epsilon W \cos \phi_1, \tag{6}$$

$$\frac{F}{W} = \sin \phi_1 + \epsilon \cos \phi_1 + \frac{1}{g}\frac{dv}{dt}. \tag{7}$$

Call F/W the specific reaction force, following Sänger, and express the change in density of the air by Hohmann's formula:

$$\frac{\rho}{\rho_0} = \left[1 - \frac{h}{400000}\right]^{49} \text{ for } h \text{ in } m,$$

$$\frac{\rho}{\rho_0} = \left[1 - \frac{h}{1312332}\right]^{49} \text{ for } h \text{ in ft.} \tag{8}$$

Using (8) and keeping in mind that $W_0 = \frac{1}{2}\rho_0 S c_L v_0^2$, $h = s \sin \phi_1$, one

obtains

$$\frac{W}{W_0}\cos\phi_1 = \left(\frac{v}{v_0}\right)^2\left[\,1 - \frac{s\sin\phi_1}{1312332}\right]^{49}. \tag{9}$$

Also accepting from Sänger that W is a constant equal to the mean value $\frac{1}{2}(W_0 + W_1)$, then

$$\frac{W}{W_0} = \frac{W_0 + W_1}{2W_0} = k_1, \qquad \frac{W_1}{W_0} = 2k_1 - 1, \tag{10}$$

After the transformation of (9) one has:

$$v = \frac{ds}{dt} = v_0\sqrt{k_1\cos\phi_1}\left[\,1 - \frac{s\sin\phi_1}{1312332}\right]^{-24.5}. \tag{11}$$

From (11) with the initial conditions $t = 0$, $s = 0$:

$$t = \frac{51500}{v_0\sin\phi_1\sqrt{k_1\cos\phi_1}}\left[\,1 - \left(1 - \frac{s\sin\phi_1}{1312332}\right)^{25.5}\right], \tag{12}$$

$$s = \frac{1312332}{\sin\phi_1}\left[\,1 - \left(1 - \frac{v_0 t\sin\phi_1\sqrt{k\cos\phi_1}}{51500}\right)^{1/25.5}\right]. \tag{13}$$

Substitutions of (11) into (12) and (13) into (11) give:

$$t = \frac{51500}{v_0\sin\phi_1\sqrt{k_1\cos\phi_1}}\left[\,1 - \left(\frac{v_0\sqrt{k_1\cos\phi_1}}{v}\right)^{1.04}\right]$$

$$\cong \frac{51500}{v_0\sin\phi_1\sqrt{k_1\cos\phi_1}}\left[\,1 - \frac{v_0\sqrt{k_1\cos\phi_1}}{v}\right], \tag{14}$$

$$v = v_0\sqrt{k_1\cos\phi_1}\left[\,1 - \frac{v_0 t\sin\phi_1\sqrt{k_1\cos\phi_1}}{51500}\right]^{-0.96}. \tag{15}$$

Keeping in mind that $\frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = \frac{dv}{ds}v$ one finds:

$$\frac{dv}{dt} = \frac{v_0^2 k_1\sin 2\phi_1}{106800}\left[\,1 - \frac{s\sin\phi_1}{1312332}\right]^{-50}. \tag{16}$$

From (1), (2)

$$k = \frac{g}{c(\alpha + 1) - \alpha v}\frac{F}{W}. \tag{17}$$

Substitution of (7), (13), (15), (16) into (17) and (17) into (4) gives the

expression for the mass ratio at the end of climbing:

$$\frac{W_1}{W_0} = e^{-\int_0^{t_1} \frac{g[\sin\phi_1 + \epsilon\cos\phi_1 + A]}{c(\alpha+1) - B}\,dt}, \tag{18}$$

$$A = \frac{v_0^2 k_1 \sin 2\phi_1}{106800\,g}\left(1 - \frac{v_0 t \sin\phi_1 \sqrt{k_1\cos\phi_1}}{51500}\right)^{-1.96}, \tag{18a}$$

$$B = \alpha v_0 \sqrt{k_1\cos\phi_1}\left(1 - \frac{v_0 t \sin\phi_1 \sqrt{k_1\cos\phi_1}}{51500}\right)^{-0.96}. \tag{18b}$$

Substitution of (14), (16) into (18) gives another formula for the mass ratio:

$$\frac{W_1}{W_0} = e^{-\int_{v_{i0}}^{v_1} \frac{g[\sin\phi_1 + \epsilon\cos\phi_1 + C]D}{[c(\alpha+1) - \alpha v]E}\,dv}, \tag{19}$$

$$C = \frac{v_0^2 k_1 \sin 2\phi_1}{106800\,g}\left(\frac{v}{v_0\sqrt{k_1\cos\phi_1}}\right)^{2.04}, \tag{19a}$$

$$D = \left(1 - \frac{s\sin\phi_1}{1312332}\right)^{50}, \qquad E = \frac{v_0^2 k_1 \sin 2\phi_1}{106800} \tag{19b}$$

where v_{i0} is the initial velocity with which the climbing begins at the zero level. Differentiation of (14) with respect to v, comparison with (16) and substitution of the obtained result for the ratio D/E into (19) gives another form:

$$\frac{W_1}{W_0} = e^{-\int_{v_{i0}}^{v_1} \frac{g[\sin\phi_1 + \epsilon\cos\phi_1 + C]G}{c(\alpha+1) - \alpha v}\,dv}, \tag{20}$$

$$G = \frac{53560}{\sin\phi_1 v^{2.04}}\,(v_0\sqrt{k_1\cos\phi_1})^{0.04}. \tag{20a}$$

Formula (20) may be simplified with a negligible effect on the final result by assuming $v^{2.04} \cong v^2$, $(v_0\sqrt{k_1\cos\phi_1})^{0.04} \cong 1$, $\frac{53560}{53400} \cong 1$, and obtain:

$$\frac{W_1}{W_0} = e^{-\int_{v_{i0}}^{v_1} \left\{\frac{53560\,g(\sin\phi_1 + \epsilon\cos\phi_1)}{[c(\alpha+1) - \alpha v]v^2\sin\phi_1} + \frac{1}{c(\alpha+1) - \alpha v}\right\} dv}. \tag{21}$$

This integral can be evaluated by elementary methods and the result is:

$$\frac{W_1}{W_0} = e^{-\frac{53560\,g(\sin\phi_1 + \epsilon\cos\phi_1)}{\sin\phi_1} H - Z}, \tag{22}$$

$$H = \log\left[\frac{c(\alpha+1)v_1 - \alpha v_{i0} v_1}{c(\alpha+1)v_{i0} - \alpha v_{i0} v_1}\right]^{\frac{\alpha}{c^2(\alpha+1)^2}} + \frac{v_1 - v_{i0}}{c(\alpha+1)v_{i0}v_1}, \tag{22a}$$

$$Z = \log\left[\frac{c(\alpha+1) - \alpha v_{i0}}{c(\alpha+1) - \alpha v_1}\right]^{1/\alpha}. \tag{22b}$$

At the end of the climbing period there is from (11):

$$v_1 = v_0\sqrt{k_1 \cos \phi_1}\left[1 - \frac{s_1 \sin \phi_1}{1312332}\right]^{-24.5}, \tag{23}$$

$$l_1 = s_1 \cos \phi_1.$$

(b) *Supersonic velocity.*

The lift and drag coefficients in a motion with supersonic velocity are given by complicated functions of some parameters. But as Sänger showed one may express the lift coefficient variation in a first, very rough approximation by the formula:

$$c_{LS} = \frac{165300}{v^2} + 0.01 \text{ for } v \text{ m./sec.* or } c_{LS} = \frac{1778000}{v^2} + 0.01 \text{ for } v \text{ ft./sec.} \tag{25}$$

Assume at the beginning of the period the values W_1, v_{i1}, s_1 and at the end W_2, v_2, s_2. The flight path is a straight line at the angle ϕ_2 to the horizontal. The procedure is quite similar to that given above. Application of the expressions:

$$W_1 = \tfrac{1}{2}\rho_0 c_L S v_{10}^2, \qquad \frac{W}{W_1} = \frac{W_1 + W_2}{2W_1} = k_2 \tag{26}$$

gives:

$$\frac{W}{W_1}\cos \phi_2 = \frac{v^2}{v_{10}^2 c_L}\left[\frac{1778000}{v^2} + 0.01\right]\left[1 - \frac{s \sin \phi_2}{1312332}\right]^{49}. \tag{27}$$

Calculate v from (27), put $v = ds/dt$ and obtain:

$$dt = \frac{ds}{\left[100 v_{10}^2 c_L k_2 \cos \phi_2 \left(1 - \frac{s \sin \phi_2}{1312332}\right)^{-49} - 177800000\right]^{1/2}}. \tag{28}$$

This is a hyperelliptic integral of 49th order which cannot be solved in a rigorous way by methods available in present mathematics, and an approximate solution has to be found. Let:

$$\bar{A} = 100 v_{10}^2 c_L k_2 \cos \phi_2, \qquad \bar{B} = 177800000, \qquad \bar{C} = -\frac{1312332}{\sin \phi_2}, \tag{28a}$$

$$1 - \frac{s \sin \phi_2}{1312332} = 1 + \frac{s}{\bar{C}} = X,$$

$$dt = \frac{\bar{C}\, dx}{\sqrt{\bar{A} X^{-49} - B}} = \frac{\bar{C}}{\sqrt{\bar{A}}} X^{49/2}\left(1 - \frac{\bar{B}}{\bar{A}} X^{49}\right)^{-1/2} dx. \tag{29}$$

* In one of his subsequent papers (Reference 3) Sänger uses also the formula: $c_{LS} = 0.04 + 1.43\, a^2/v^2 = 0.04 + 165300/v^2$ where a = velocity of sound in m./s. It is obvious that application of this formula will not introduce any substantial change in the derived method. Several numerical coefficients only will be subject to change.

Apply the binomial theorem to (29), and since it is not probable that in present considerations greater heights than 500,000 ft. will be taken into account, one sees that X will always be less than one half. Thus the series obtained is a fairly rapidly converging one. Integrating term by term between the limits t_1 and t, s_1 and s:

$$t - t_1 = \frac{131233.2}{v_{10} \sin \phi_2 \sqrt{c_L k_2 \cos \phi_2}} \left\{ \frac{1}{25.5} \left[\left(1 - \frac{s_1 \sin \phi_2}{1312332} \right)^{25.5} - \left(1 - \frac{s \sin \phi_2}{1312332} \right)^{25.5} \right] + \frac{B}{149A} \left[\left(1 - \frac{s_1 \sin \phi_2}{1312332} \right)^{74.5} - \left(1 - \frac{s \sin \phi_2}{1312332} \right)^{74.5} \right] + \frac{3B^2}{988A^2} \left[\left(1 - \frac{s_1 \sin \phi_2}{1312332} \right)^{123.5} - \left(1 - \frac{s \sin \phi_2}{1312332} \right)^{123.5} \right] + \ldots \right\}. \quad (30)$$

Since the powers in this expression rise rapidly, one may for comparative purposes take only the first term into account:

$$s = \frac{1312332}{\sin \phi_2} \left\{ 1 - \left[\left(1 - \frac{s_1 \sin \phi_2}{1312332} \right)^{25.5} - \frac{v_{10} \sin \phi_2 (t - t_1) \sqrt{c_L k_2 \cos \phi_2}}{5140} \right]^{1/25.5} \right\}. \quad (31)$$

Put:

$$1 - \frac{s_1 \sin \phi_2}{1312332} = L. \quad (31a)$$

Substitution of the value for X from (27) into (30) and of (31) into the formula for v from (27) gives:

$$t - t_1 = \frac{5140}{v_{10} \sin \phi_2 \sqrt{c_L k_2 \cos \phi_2}} \left[L^{25.5} - \left(\frac{\bar{A}}{v^2 + \bar{B}} \right)^{0.5204} \right], \quad (32)$$

$$v = \left\{ \bar{A} \left[L^{25.5} - \frac{v_{10}(t - t_1) \sin \phi_2 \sqrt{k_2 c_L \cos \phi_2}}{5140} \right]^{-1.921} - \bar{B} \right\}^{1/2}. \quad (33)$$

In the same way as in the previous chapter one finds:

$$\frac{dv}{dt} = \frac{1225 v_{10}^2 c_L k_2 \sin 2\phi_2}{1312332} X^{-50} \quad (34)$$

Substitutions of (7) with ϕ_2, (31), (33), (34) into (17) and (17) into

(4) with W_1 and W_2 instead of W_0 and W_1 give:

$$\frac{W_2}{W_1} = e^{-\int_{t_1}^{t_2} \frac{g\left\{\sin\phi_2+\epsilon\cos\phi_2+\frac{1225 v_{10}^2 c_L k_2 \sin 2\phi_2}{1312332\, g}\,[L^{25.5}-K]^{-1.96}\right\}}{c(\alpha+1)-\alpha\{\bar{A}\,[L^{25.5}-K]^{-1.921}-\bar{B}\}^{1/2}}\,dt}, \tag{35}$$

$$K = \frac{v_{10}(t-t_1)\sin\phi_2\sqrt{k_2 c_L\cos\phi_2}}{5140}. \tag{35a}$$

(35) may be transformed. Differentiating (32) with respect to v, and using the substitutions $z^{0.5204} \cong z^{0.5}$,

$$dt = \frac{7250000\, v}{\sin\phi_2(v^2+177800000)^{1.5}}\,dv. \tag{36}$$

From (33) obtain:

$$[L^{25.5}-K]^{-1.96} = \left[\frac{v^2+\bar{B}}{\bar{A}}\right]^{1.0203}. \tag{36a}$$

Substitution of (36) and (36a) into (35) with the assumption that $z^{1.0203} \cong z$ leads to:

$$\frac{W_2}{W_1} = e^{-\int_{v_{11}}^{v_2} \frac{7250000\, gv\left[\sin\phi_2+\epsilon\cos\phi_2+\frac{49\sin\phi_2}{2624664}(v^2+177800000)\right]}{[c(\alpha+1)-\alpha v]\sin\phi_2(v^2+177800000)^{1.5}}\,dv}. \tag{37}$$

This is a hyperelliptic integral of 6th order. Methods exist which enable the solution of integrals of this order by use of a transformation to elliptic integrals. As is known, the evaluation of these integrals usually takes many pages of calculations. If one wants to avoid the evaluation of this integral by advanced methods, one may evaluate it in any particular case by a graphical procedure. In order not to obscure the calculations by mathematical derivations, the author leaves this form of the integral. The horizontal distance covered in this period of flight is equal to:

$$l_2 = (s_2 - s_1)\cos\phi_2. \tag{38}$$

2. *Climbing of a Rocket-Plane.*

(a) *Subsonic velocity.*

For this type of plane all the formulae are obtained by substitution of $\alpha = 0$ in the equations derived above. Thus the integration of (18) give:

$$\frac{W_1}{W_0} = e^{-\frac{gt_1}{c}(\sin\phi_1+\epsilon\cos\phi_1)-\frac{0.962}{0.96}\frac{v_0}{c}\sqrt{k_1\cos\phi_1}\,M}, \tag{39}$$

$$M = \left[\left(1-\frac{v_0\sin\phi_1 t_1\sqrt{k_1\cos\phi_1}}{51500}\right)^{-0.96}-1\right]. \tag{39a}$$

This was the only equation derived by Sänger. The integration of (21) gives:

$$\frac{W_1}{W_0} = e^{\frac{53560\, g(\sin\phi_1 + \epsilon\cos\phi_1) + v_1 v_{10}\sin\phi_1}{c\sin\phi_1\, v_1 v_{10}}(v_{10} - v_1)}. \tag{40}$$

(b) *Supersonic velocity.*

Substitution of $\alpha = 0$ into (35) and integration with the assumption that $z^{0.96} \cong z$ give:

$$\frac{W_2}{W_1} = e^{-\frac{g}{c}(\sin\phi_2 + \epsilon\cos\phi_2)(t_2 - t_1) - \frac{10 v_{10}\sqrt{c_L k_2}\cos\phi_2}{c} N}, \tag{41}$$

$$N = \left\{ \left[L^{25.5} - \frac{v_{10}(t_2 - t_1)\sin\phi_2 \sqrt{k_2 c_L}\cos\phi_2}{5140} \right]^{-1} - L^{-25.5} \right\}. \tag{41a}$$

Another formula may be obtained by substitution of $\alpha = 0$ into (37).

3. *Horizontal Flight of a Jet-Plane.*

(a) *Subsonic velocity.*

The formulae in this period of flight are easy to derive from those given above by use of the substitutions $h = \text{const.}$, $\phi = 0$. At the beginning of this period one has the values W_2, s_2, t_2 and at the end W_3, s_3, t_3. Apply the expressions:

$$W_2 = \tfrac{1}{2}\rho_0 c_L S v_{02}^2, \qquad \frac{W}{W_2} = k_3, \qquad v = v_{02}\sqrt{k_3}. \tag{42}$$

The assumption of constant average weight during the whole period of flight leads to the result that flight is with a uniform velocity.

$$t - t_2 = \frac{s - s_2}{v_{02}\sqrt{k_3}}, \tag{42a}$$

$$\frac{W_3}{W_2} = e^{-\frac{g\epsilon(t_3 - t_2)}{c(\alpha+1) - \alpha v_{02}\sqrt{k_3}}}. \tag{43}$$

The horizontal distance covered is $l_3 = (s_3 - s_2)$.

(b) *Supersonic velocity.*

In flight at great heights with supersonic velocity the centrifugal force may be taken into account. Again use the expressions:

$$W_3 = \tfrac{1}{2}\rho_0 c_L S v_{03}^2, \qquad \frac{W}{W_3} = e^{-\int_{t_3}^{t} k\, dt} = k^4. \tag{44}$$

One must take into consideration all forces acting on the plane:

lift:

$$\frac{L}{W_3} = \frac{v^2}{v_{03}^2 c_L}\left[\frac{1778000}{v^2} + 0.01\right], \tag{45}$$

centr. force:

$$\frac{F_c}{W_3} = \frac{v^2 W}{g(R+h)W_3} = \frac{v^2}{g(R+h)}\, e^{-\int_{t_3}^{t} k\, dt}, \tag{46}$$

react. force:

$$\frac{F}{W_3} = \left[\frac{kc}{g} + \frac{\alpha k(c-v)}{g}\right] e^{-\int_{t_3}^{t} k\, dt}, \tag{47}$$

drag:

$$\frac{D}{W_3} = \frac{\epsilon v^2}{v_{03}^2 c_L}\left[\frac{1778000}{v^2} + 0.01\right], \tag{48}$$

inertia force:

$$\frac{T}{W_3} = \frac{1}{g}\, e^{-\int_{t_3}^{t} k\, dt}\frac{dv}{dt}. \tag{49}$$

Besides these the weight W has to be taken into account. The conditions of equilibrium of all forces in the radial and tangential directions give:

$$\left[\frac{v^2}{g(R+h)} - 1\right] k_4 + \frac{v^2}{v_{03}^2 c_L}\left[\frac{1778000}{v^2} + 0.01\right] = 0, \tag{50}$$

$$\frac{k}{g}[c + \alpha(c-v)]k_4 - \frac{\epsilon v^2}{v_{03}^2 c_L}\left[\frac{1778000}{v^2} + 0.01\right] - \frac{1}{g} k_4 \frac{dv}{dt} = 0. \tag{51}$$

Calculate from (50) the value of v and from (51) the value of k and substitute it into (44):

$$v^2 = \frac{100g(R+h)[v_{03}^2 c_L k_4 - 1778000]}{100 c_L k_4 v_{03}^2 + g(R+h)} = \text{const.}, \tag{52}$$

$$\frac{W_4}{W_3} = e^{-\frac{g\epsilon(t_4 - t_3)}{[c(\alpha+1) - \alpha v]k_4}\left[\frac{1778000}{v_{03}^2 c_L} + \frac{0.01 v^2}{v_{03}^2 c_L}\right]}, \tag{53}$$

$$(s - s_3) = v(t - t_3), \qquad l_4 = (s_4 - s_3). \tag{54}$$

4. *Horizontal Flight of a Rocket-Plane.*

All equations are the same as for a jet-plane with $\alpha = 0$.

5. *Gliding.*

(a) *Supersonic velocity.*

During gliding there is no reactional force, and the weight of the plane remains the same. In case the flight path is a curved line and the velocity of flight is very great, the centrifugal force may be taken into account. As the initial values for this period assume t_4, v_{i4}, s_4 and

the end values t_5, v_5, s_5. Consider the forces acting on the plane:

$$L = \tfrac{1}{2}\rho_0 S\left(\frac{1778000}{v^2} + 0.01\right)\left(1 - \frac{h}{1312332}\right)^{49} v^2, \tag{55}$$

$$F_c = \frac{W_4 v^2}{(R+h)g} = \frac{W_4 v^2}{g_0 R^2}(R+h), \tag{56}$$

$$D = L\epsilon, \tag{57}$$

$$T = \frac{W_4}{g_0 R^2}(R+h)^2 \frac{dv}{dt}. \tag{58}$$

The use of the substitution $\bar{k} = \rho_0 S/(2W_4) = \text{const.}$ leads to:

$$L = \bar{k} W_4\left(\frac{1778000}{v^2} + 0.01\right)\left(1 - \frac{h}{1312332}\right)^{49} v^2. \tag{59}$$

The conditions of equilibrium of forces in the radial and tangential directions give:

$$\bar{k}\left(\frac{1778000}{v^2} + 0.01\right)\left(1 - \frac{h}{1312332}\right)^{49} v^2 + \frac{v^2}{g_0 R^2}(R+H) = \cos\phi_3, \tag{60}$$

$$\epsilon\bar{k}\left(\frac{1778000}{v^2} + 0.01\right)\left(1 - \frac{h}{1312332}\right)^{49} v^2 = \sin\phi_3 + \frac{(R+h)^2}{g_0 R^2}\frac{dv}{dt}. \tag{61}$$

In (61) it is assumed that the motion is a decelerated one. The angle ϕ_3 is the instantaneous angle between the tangent to the path at any arbitrary point and a horizontal line. Eliminate $\left(1 - \frac{h}{1312332}\right)^{49}$ from (60) and (61) and:

$$\frac{d^2 s}{dt^2} = \frac{dv}{dt} = \frac{g_0 R^2}{(R+h)^2}\cos\phi_3 - g_0 \sin\phi_3 \frac{R^2}{(R+h)^2} - \frac{\epsilon v^2}{R+h}. \tag{62}$$

In order to solve this equation in an elementary way, more simplifications must be introduced. The functions $\phi_3 = f(v)$ or $\phi_3 = f(t)$ and $h = f(v)$ or $h = f(t)$ present some difficulties. For this reason assume that $(R+h) \cong R$, and for ϕ_3 take a mean constant value. The first simplification has a small influence on the final result; the second is very rough but necessary. The difference between straight line flight and the flight discussed is that the term representing the centrifugal

force remains. The first integration gives:

$$t - t_4 = (2P)^{-1} ln \left[A \frac{S + vP}{S - vP} \right], \tag{63}$$

$$P = \sqrt{\frac{\epsilon}{R}} \sqrt{S}, \tag{63a}$$

$$S = g_0[\epsilon \cos \phi_3 - \sin \phi_3], \tag{63b}$$

$$A = \frac{S - v_{i4}P}{S + v_{i4}P}, \tag{63c}$$

$$v = \sqrt{\frac{R}{\epsilon}} \sqrt{S} \frac{e^{2(t-t_4)P} - A}{e^{2(t-t_4)P} + A}. \tag{64}$$

Second integration gives

$$(s - s_4) = \frac{R}{\epsilon} ln \frac{A + e^{2(t-t_4)P}}{A + 1} - \sqrt{\frac{R}{\epsilon}} \sqrt{S}(t - t_4). \tag{65}$$

The total distance covered in this period is:

$$(s_5 - s_4) = \frac{h}{\sin \phi_3}. \tag{66}$$

Knowing the height at which the velocity changes from supersonic to subsonic, one may calculate the first approximation of $(s_5 - s_4)$ for the chosen value of ϕ_3 from (66), the time from (65) and the final velocity from (64). If one assumes that the flight path is a straight line at an angle ϕ_3 to the horizontal, then the equations are very simple:

$$(v - v_{i4}) = S(t - t_4), \tag{67}$$

$$(s - s_4) = \tfrac{1}{2}S(t - t_4)^2 + v_{i4}(t - t_4). \tag{68}$$

The horizontal distance is:

$$l_5 = (s_5 - s_4) \cos \phi_3. \tag{69}$$

(b) *Subsonic velocity.*

Assume that c_L = const., that the flight path is a straight line and that the initial values are v_{i5}, s_5, t_5 and the end values v_6, s_6, t_6. The two conditions of the equilibrium of forces are:

$$\bar{k} c_L \left(1 - \frac{h}{1312332} \right)^{49} v^2 = \cos \phi_4, \tag{70}$$

$$\epsilon \bar{k} c_L \left(1 - \frac{h}{1312332} \right)^{49} v^2 = \sin \phi_4 + \frac{1}{g_0} \frac{dv}{dt}. \tag{70a}$$

The results are:

$$(v - v_{i5}) = S(t - t_5), \tag{71}$$

$$(s - s_5) = \tfrac{1}{2}S(t - t_5)^2 + v_{i5}(t - t_5), \tag{72}$$

$$l_6 = (s_6 - s_5)\cos\phi_4. \tag{73}$$

Another form of the approximate calculation of horizontal distance is:

$$l_5 = \frac{K_4}{D} = \frac{W_4 v_{i4}^2}{2\epsilon W_4} = \frac{v_{i4}^2}{2\epsilon} \tag{74}$$

where K_4 is the kinetic energy at the end of the horizontal flight. Assuming for D or ϵ a mean value during gliding, one may obtain the approximate horizontal distance.

6. *Optimum Height of Transition from Subsonic to Supersonic Velocity.*

Assume a horizontal flight in the range of the subsonic velocity with c_L independent of the velocity, i.e. $c_L = \text{const}$. Then

$$v^2 = v_0^2\left[1 - \frac{h}{1312332}\right]^{-49} \tag{75}$$

where v is the velocity at a certain height and v_0 is the velocity at sea level for the same weight in both cases. Similarly assume a horizontal flight with the supersonic velocity and take the centrifugal force into account. Then:

$$W = \left[1 - \frac{h}{1312332}\right]^{49}\left[\frac{1778000}{v^2} + 0.01\right]\frac{Wv^2}{c_L v_0^2} + \frac{Wv^2}{g_0 R^2}(R + h). \tag{76}$$

Introduce the substitution $p = 1/(c_L v_0^2)$ and obtain from (75) and (76): subsonic:

$$1 = pc_L v^2\left[1 - \frac{h}{1312332}\right]^{49}, \tag{77}$$

supersonic:

$$1 = p\left[1 - \frac{h}{1312332}\right]^{49}\left[1778000 + 0.0\,v^2\right] + \frac{v^2}{g_0 R^2}(R + h). \tag{78}$$

At the transition height both equations must give the same velocity. Assume $(R + h) \cong R$, compare (77) and (78), calculate $\left[1 - \frac{h}{1312332}\right]^{49}$ and in the next step calculate h. In the case where one neglects the

centrifugal force, the result is very simple:

$$\left[1 - \frac{h}{1312332}\right]^{49} = \frac{c_L - 0.01}{1778 c_L}. \tag{79}$$

The table below gives the values of h for $p = 0.001$ and various values of c_L. Sänger solved this problem in a graphical way by intersection of two curves and obtained for the same value of p the value of $h \cong 131000$ ft. and $v = \sim 1740$ ft./sec.

c_L	h ft.	v ft./sec.
1.0	186,000	1338
0.1	187,600	4435
0.08	188,300	5010
0.04	190,000	7690
0.01	1,312,332	∞

If the jet plane taking air from outside is considered, one has to keep in mind that the height of flight will be limited by the oxygen content of the air. The table below gives the percentage of oxygen in the air:

h ft.	0	65,616	131,232	196,848	262,464
per cent. O in volume	20.9	15	10	6	1

7. *Average Coefficients of Efficiency.*

(a) *Climbing with subsonic velocity.*

Take into account only the first and the last moments in each period. The increase in kinetic energy of the weight W_1 is:

$$K_1 = \frac{W_1}{2g}(v_1^2 - v_{i0}^2), \tag{80}$$

of potential energy:

$$E_1 = W_1 h_1 = W_1 s_1 \sin \phi_1. \tag{81}$$

The energy due to the work done during this period is

$$M_1 = D s_1 = \epsilon W \cos \phi_1 s_1 = \epsilon W_0 k_1 l_1, \ \phi_1 \neq 90^\circ \tag{82}$$

and the energy content of the burned fuel is

$$Q_1 = (W_0 - W_1)Q = \frac{1}{2g}(W_0 - W_1)c_t^2 \tag{83}$$

where Q is the number of kilocalories per pound of fuel. c_t, in the case of combustibles possessing oxygen in themselves, is the maximum theoretical velocity of the exhaust gases. In the case of combustibles

not possessing oxygen in themselves, it is an imaginary value obtained by introducing the mechanical equivalent of heat and equating the heat content to the potential energy or kinetic energy with the velocity c_t. The lost energy of the plane is

$$L_1 = \frac{1}{2g}(W_0 - W_1)v_{i0}^2. \tag{84}$$

The ratio

$$\eta_1 = \frac{E_1 + K_1 + M_1}{Q_1 + L_1} \tag{85}$$

is called the average coefficient of efficiency during the climbing period. Substitute in (85) the formulae (80) to (84) and (10):

$$\eta_1 = \frac{2g(2k_1 - 1)s_1 \sin\phi_1 + 2g\epsilon l_1 k_1 + (2k - 1)(v_1^2 - v_{i0}^2)}{(2 - 2k_1)(c_t^2 + v_{i0}^2)}. \tag{86}$$

In the purely theoretical case that k_1 approaches its minimum value of one-half, assume $v_{i0} = 0$ whence η_1 approaches:

$$\eta_1 = \frac{g\epsilon l_1}{c_t^2} = \frac{g\epsilon s_1 \cos\phi_1}{c_t^2}. \tag{87}$$

Calculating from (23) and substituting into (87):

$$\eta_1 = \frac{1312332g\epsilon \cos\phi_1 T}{c_t^2 \sin\phi_1}, \tag{88}$$

$$T = 1 - \left(\frac{v_0\sqrt{k_1 \cos\phi_1}}{v_1}\right)^{1/24.5}. \tag{88a}$$

Assume for illustrative purposes $v_0/v_1 = 1/5$, $\phi_1 = 30°$, $\epsilon = 0.2$. Thus $\eta_1 \cong (1094000/c_t^2)$. The low average value of c_t is of order of 10,000 ft./sec. For this value $\eta_1 \cong 1.094$ per cent. As we see, the low average coefficient of efficiency is inherent in climbing flight. Sänger in one of his numerical examples with the initial velocity equal to about 260 ft./sec. obtained $\eta_1 \cong 3.08$ per cent., so the value is of the same order.

(b) *Climbing with supersonic velocity.*

The gain in kinetic energy of the weight W_2 is:

$$K_2 = \frac{W_2}{2g}(v_2^2 - v_{i1}^2), \tag{89}$$

the gain in the potential energy:

$$E_2 = W_2(s_2 - s_1)\sin\phi_2, \tag{90}$$

the energy due to the work done:

$$M_2 = D(s_2 - s_1) = \epsilon W_1 k_2 l_2,\ l_2 \neq 0,\ \epsilon \neq \infty, \tag{91}$$

the loss in energy contents of the fuel:

$$Q_2 = \frac{1}{2g}(W_1 - W_2)c_t^2, \tag{92}$$

the loss in energy of the plane:

$$L_2 = \frac{1}{2g}(W_1 - W_2)v_{i1}^2, \tag{93}$$

$$\eta_2 = \frac{(2k_2 - 1)[2g(s_2 - s_1)\sin\phi_2 + (v_2^2 - v_{i1}^2)] + 2g\epsilon k_2 l_2}{(2 - 2k_2)(c_t^2 + v_{i1}^2)}. \tag{94}$$

For $k_2 \to \frac{1}{2}$, η_2 approaches

$$\eta_2 = \frac{g\epsilon l_2}{c_t^2 + v_{i1}^2}. \tag{95}$$

(c) *The coefficient of efficiency for the whole period of climbing.*

$$K_c = \frac{W_2}{2g}(v_2^2 - v_{i0}^2), \qquad E_c = W_2[s_1 \sin\phi_1 + (s_2 - s_1)\sin\phi_2], \tag{96a}$$

$$M_c = \epsilon k_1 W_0 l_1 + \epsilon k_2 W_1 l_2, \qquad Q_c = \frac{W_0 - W_2}{2g}c_t^2, \tag{96b}$$

$$L_c = \frac{1}{2g}(W_0 - W_2)v_{i0}^2, \qquad \frac{W_2}{W_0} = (2k_2 - 1)(2k_1 - 1), \tag{96c}$$

$$\eta_c = \frac{(W_2/W_0)\cdot U + V}{[1 - W_2/W_0](c_t^2 + v_{i0}^2)}, \tag{97}$$

$$U = 2g[s_1 \sin\phi_1 + (s_2 - s_1)\sin\phi_2] + v_2^2 - v_{i0}^2, \tag{97a}$$

$$V = 2g\epsilon[k_1 l_1 + k_2(2k_1 - 1)l_2]. \tag{97b}$$

(d) *Horizontal flight.*

Subsonic velocity:

$$K_3 = \frac{W_3}{2g}(v_3^2 - v_{i2}^2), \qquad M_3 = \epsilon k_3 W_2(s_3 - s_2), \tag{98a}$$

$$Q_3 = \frac{W_2 - W_3}{2g}c_t^2, \qquad L_3 = \frac{1}{2g}(W_2 - W_3)v_{i2}^2, \tag{98b}$$

$$\eta_3 = \frac{(2k_3 - 1)(v_3^2 - v_{i2}^2) + 2g\epsilon k_3(s_3 - s_2)}{(2 - 2k_3)(c_t^2 + v_{i2}^2)}. \tag{99}$$

For $k_3 \to \frac{1}{2}$

$$\eta_3 = \frac{g\epsilon(s_3 - s_2)}{c_t^2 + v_{i2}^2}. \tag{99a}$$

But from (43) it is evident that for $k_3 \to \frac{1}{2}$, and for finite values of v_{02}, c, α the value $(s_3 - s_2) \to \infty$ which practically means that η_3 may reach high values.

Supersonic velocity:

$$\eta_4 = \frac{(2k_4 - 1)(v_4^2 - v_{i3}^2) + 2g\epsilon k_4(s_4 - s_3)}{(2 - 2k_4)(c_t^2 + v_{i3}^2)}. \tag{100}$$

Total horizontal flight:

$$\eta_t = \frac{(W_4/W_2)\cdot(v_4^2 - v_{i2}^2) + 2g\epsilon[k_3(s_3 - s_2) + k_4(2k_3 - 1)(s_4 - s_3)]}{(1 - W_4/W_2)(c_t^2 + v_{i2}^2)}, \tag{101}$$

$$\frac{W_4}{W_2} = (2k_4 - 1)(2k_3 - 1). \tag{101a}$$

(e) *The average coefficient of efficiency for the whole propelled period.*

$$\eta_t = \frac{2g\left(\frac{W_4}{W_0}\right)[s_1 \sin\phi_1 + (s_2 - s_1)\sin\phi_2] + \left(\frac{W_4}{W_0}\right)(v_4^2 - v_{i0}^2) + X}{(1 - W_4/W_0)(c_t^2 + v_{i0}^2)}, \tag{102}$$

$$X = k_1 s_1 \cos\phi_1 + \gamma(s_2 - s_1)\cos\phi_2 + \delta(s_3 - s_2) + \lambda(s_4 - s_3), \tag{102a}$$

$$\gamma = k_2(2k_1 - 1), \qquad \delta = k_3(2k_2 - 1)(2k_1 - 1), \tag{102b}$$
$$\lambda = k_4(2k_3 - 1)(2k_2 - 1)(2k_1 - 1).$$

In all cases: $L \neq 0$, $\epsilon \neq \infty$, $\phi_{1,2} \neq 90°$.

FINAL REMARKS.

Some remarks may be made regarding the graphical representation of the results obtained. For the assumed value of k_i, which may depend upon the design requirements, one may plot the values F/W, v, s_1, dv/dt as functions of time for each period of flight. By evaluating the integrals by which the values of W_i/W_{i-1} are expressed, one obtains the final value of t or v. Thus all parameters may be presented graphically. For the gliding period it may sometimes be better to express the parameters t and v as functions of the height. Some diagrams of this type for a rocket plane, but for flight paths quite different from those discussed in the present paper, may be found in Reference 1. By calculation of all of these factors for various values of the parameter

k_i, one may obtain for each period of flight a graph representing the potential energy, the kinetic energy, the work done, the height reached, the loss in energy content of the fuel, the horizontal distance, and the coefficient of efficiency at the end of the period as functions of k_i, the total time, or of the end velocity. Such a diagram may be obtained for the whole flight thus representing the final result.*

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THE ELIMINATION OF COMBUSTION KNOCK.

BY

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"The study of these (heat) engines is of the greatest interest, their importance is enormous, their use is increasing continually, and they seem destined to produce a great revolution in the civilized world." . . . Reflexions sur la Puissance Motrice de Feu . . . S. Carnot, 1824.

A revolution indeed, . . . there are few acts of modern life that do not depend quite directly on the use of power, and the heat engines that produce much of that power have become a foundation of our civilization. Because of its efficiency, convenience and safety, the reciprocating internal-combustion engine is the most important single type of heat engine in use today, and most likely it will continue so for many years. In the United States alone more than two hundred forty million horsepower of internal-combustion engines and twenty-six billion gallons of internal-combustion engine fuels were produced during 1940.

Reciprocating internal-combustion engines are built in two main types; spark ignition engines operating on the Otto-cycle, and compression ignition engines operating on the Diesel cycle. From their wide spread use as the source of motive power for automobiles and aircraft, it has become popular knowledge that annoying and destructive combustion phenomena, known as knock and preignition, are liable to occur in Otto-cycle engines. To operate these engines at desirable levels of power and efficiency requires that they be supplied with fuels having relatively high resistance to knock and preignition (high Octane Number[3]). It is less well known that the solid injection Diesel engine also will knock if it is not supplied with a fuel that ignites rather easily (high Cetane Number[4]). The development and volume supply of fuels having useful values of these special ignition qualities has paced the technical development on which wide scale utilization of the internal-combustion engine depends. To a considerable extent the further technical development of internal-combustion engines appears to depend, either on further development of these special fuel ignition qualities or on the development of means for carrying out the combustion process independent of these qualities.

This paper presents a review of the origin of fuel ignition quality requirements and from this it develops a plan for carrying out the

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[3] See ASTM Standard Method of Test on Knock Characteristics of Motor Fuels D357-45.

[4] See ASTM Standard Method of Test for Ignition Quality of Diesel Fuels D613-43T.

combustion process in an internal-combustion engine so that it is independent of the ignition quality of the fuel used. The plan is illustrated with results obtained from the operation of single cylinder engines under laboratory conditions.

The relative merits of basing further internal-combustion engine developments on continuing the improvement of fuel ignition qualities or, on developing means for carrying out the combustion independent of ignition quality is inappropriate for discussion at this time. However, there are two important points in favor of the latter procedure that appear to deserve mention:

(*a*) In refining naturally occurring crude oil to produce fuels of desirably high Octane Number and Cetane Number, losses occur such that the resulting quantity of fuel per barrel of crude oil is less than it would have been had it not been necessary to produce the fuels with high Octane or Cetane Number. This is an important consideration in the conservation of petroleum reserves.

(*b*) In addition to the processing losses, there are direct costs for selection and processing of the crude petroleum such that fuels made to meet desirable Octane Number and Cetane Number specifications cost more than fuels without those specification requirements.

ORIGIN OF IGNITION QUALITY REQUIREMENTS.

In developing the present working knowledge of the internal-combustion engine, it has been important to establish rather definite ideas about how and why knock and preignition occur and give rise to the requirements for special fuel ignition quality characteristics. The researches on this aspect of the problem have been too numerous to cite in detail but they combine to form reasonably clear pictures which are appropriate to review as a basis for presenting the scheme for carrying out the combustion process independent of ignition quality.

Otto-Cycle Engine, Octane Requirement.

In the spark ignition Otto-cycle engine, the cylinder contents comprise a more or less uniform mixture of air and fuel vapor which is under relatively high pressure at the time it is ignited by the spark. When ignition occurs, a flame front is established which progresses into the combustible mixture, compressing ahead of it the portion of the air-fuel vapor mixture that is yet to be burned. This situation is illustrated by Fig. 1.[5] The air-fuel vapor mixture that will be burned last, the "end gas," is compressed by the forward progress of the flame front. At the high levels of temperature and pressure resulting from this compression, oxidation reactions tend to occur quite rapidly and may lead to sponta-

[5] See National Advisory Committee for Aeronautics Report No. 704, dated 1941. Also, NACA Technical Film No. 13 which gives an extremely clear photographic description of this process.

neous ignition of the end gas which is identified as knock. The end gas reactions almost invariably would lead to spontaneous ignition or knock at the pressure and temperature levels occurring in practical engine operations if they were given sufficient time. However, the end gas may be consumed by the normal progress of the flame front before it ignites spontaneously and, in this event, no knock occurs. Thus, the occurrence or non-occurrence of knock may be likened to a race between the end gas reactions and the normal progress of the flame front. Fuels of high octane number have high resistance to the occurrence of

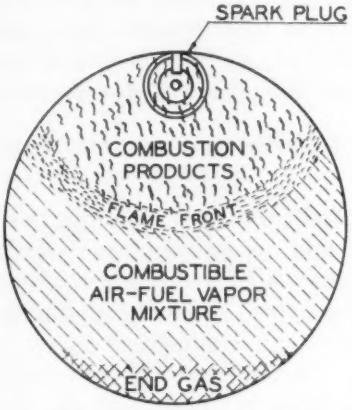


FIG. 1. Schematic diagram of the normal combustion of a preformed mixture showing the "end gas" being compressed ahead of the advancing flame front.

spontaneous ignition and permit the normal progress of the flame front to win the race, even at high levels of temperature and pressure, i.e., at high manifold pressure, intake temperature, compression ratio, spark advance, etc.

Otto-cycle Engine, Preignition.

Preignition is the term used to designate the occurrence of ignition at some location of the combustion chamber or time of the cycle other than at the spark. This ignition, which is uncontrolled as to time or place, generally occurs at a point of contact of the air-fuel vapor mixture with some hot part of the combustion chamber. The liklihood for it to occur appears to depend on such conditions as the temperature of the contact point, the pressure of the air-fuel vapor mixture and the duration of contact of the mixture with the hot point. The available data do not permit generalization but there is at least a rough trend for fuels of high Octane Number to have a high resistance to preignition.

Diesel-cycle Engine, Cetane Requirement.

In a Diesel engine, with the piston near its top position on the compression stroke, fuel is injected through a nozzle into the highly com-

pressed and heated air that fills the combustion space. No ignition source is provided. The fuel spray rapidly approaches the temperature of the compressed air and reactions occur which may lead to spontaneous ignition of the initial portions of the spray. The resulting inflamation serves as a source of ignition for the remainder of the fuel. Time is required from the beginning of injection until spontaneous ignition occurs. This time is called the ignition lag and it must be held to a reasonably low value to obtain practical engine operation.

Cetane Number is a measure of the ignition tendency of a fuel under Diesel engine conditions; high Cetane Number fuels ignite readily and permit engine operation at high speeds, low temperatures, low compression ratio, etc.; low Cetane Number fuels are slow to ignite and limit engine operation to low speed, high compression ratio, etc.

COMBUSTION WITHOUT FUEL IGNITION QUALITY REQUIREMENTS.

Theoretical Case.

It will have been noted from the foregoing, that in the cases of spark ignition engine knock and preignition, fuel ignition quality requirements result from an undesired tendency for spontaneous ignition to occur; while in the case of the Diesel engine knock, the ignition quality requirement results from an undesired delay in the occurrence of spontaneous ignition. These cases of spontaneous ignition have a common characteristic, namely; that ignition, whether desired or undesired, results from the residence of combustible air-fuel vapor mixture for an appreciable time at combustion chamber temperatures and pressures.

It appears that the undesired spontaneous ignition that leads to knock or preignition, or both, in a spark ignition engine, could be eliminated if it were possible to arrange the physical conditions within the combustion chamber to prevent a combustible air-fuel vapor mixture from having an appreciable residence time at engine temperatures and pressures.

More or less as a pure speculation and quite independent of any possible physical means for accomplishing the circumstances, Fig. 2 visualizes a situation wherein end gas residence time is rendered inappreciable and the possibility for knock or preignition to occur is eliminated.

Referring to Fig. 2, four zones are noted: Combustion Products, Flame Front, Combustible Mixture and Air. It is conceived that fuel is introduced to form a combustible mixture only in a limited zone immediately ahead of the moving flame front so that each element of combustible mixture is consumed by the flame front almost immediately after it is formed. This process must be continuous, the fuel being introduced to form a combustible mixture which "leads" the flame across the combustion chamber as far as is desired.

It is difficult, perhaps, to visualize a mechanism to distribute fuel to form the continuously moving, leading zone, of combustible mixture. However, for speculative purposes, it is perfectly permissible to imagine a device which distributes fuel molecules uniformly and at a proper rate to maintain the leading zone of combustible air-fuel vapor mixture always just ahead of the flame.

The flame is quite indifferent to what exists far ahead of it and it will progress normally if a desired mixture is presented just ahead of it in its direction of burning. Thus, if our imaginary device is sufficiently well instructed so that it will deliver the first element of combustible mixture to a spark plug at the proper time and, if it leads the resulting flame across the combustion chamber with a continuous, uniform "leading mixture zone," then—with the important exception that the possi-

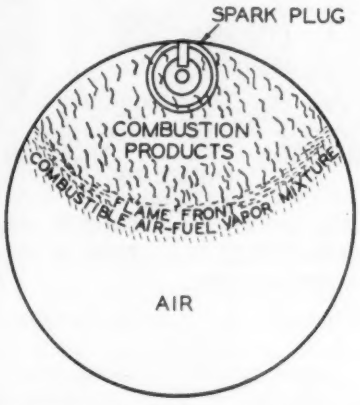


FIG. 2. Schematic diagram of hypothetical combustion process in which end gas is made vanishingly small. Combustible mixture zones of very limited extent are formed successively to lead the flame across the combustion chamber. The process is initiated by causing the first combustible mixture zone to occur at the time and place for ignition.

bility of knock and preignition is eliminated—the resulting combustion will not be significantly different from combustion in a conventional spark-ignition, Otto-cycle engine.

Approximations to the Theoretical Case.

It has not been possible to describe the idealized conditions for knock-free operation in terms of known units of apparatus that would exactly fulfill those conditions. However, a number of arrangements of real apparatus can be visualized that may approximate those conditions close enough to produce the desired results. Three of these arrangements are shown schematically in Fig. 3.

Referring to Fig. 3A: Fuel is introduced into the combustion chamber through an injection nozzle located in the center of the chamber. The first increment of injected fuel forms a localized zone "A" of com-

bustible mixture which is ignited by the spark. Subsequent injected portions of the fuel form combustible zones "B", "C", . . . successively, which lead the flame around the combustion chamber until as much of the air as may be desired has been consumed. An extension of this scheme is to use two sparks diametrically opposed and lead the flame half-way around the combustion chamber from each of them. Also, three or four or more sparks and corresponding sprays might be used.

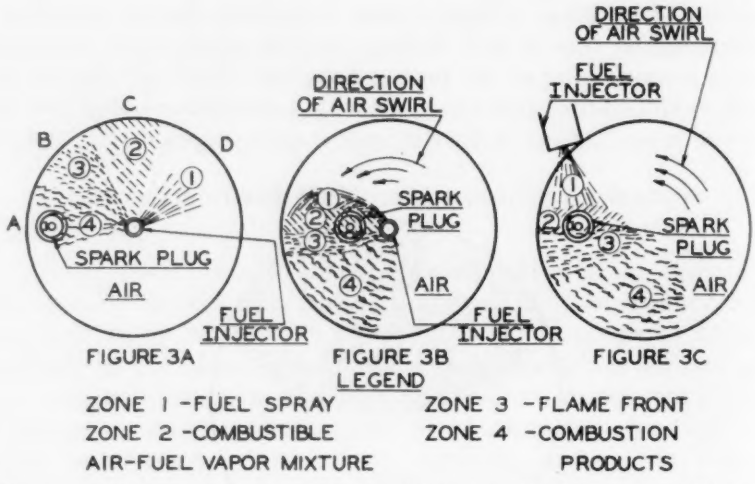


FIGURE 3A FIGURE 3B FIGURE 3C

Three possible approximations of the hypothetical combustion process of Fig. 2.

FIG. 3A. Air in the combustion chamber is more or less stationary. The first zone of combustible mixture "A" is formed from a centrally located injector about a Spark Plug at the time of sparking. Successive zones "B", "C", . . . are formed to lead the flame around the combustion chamber.

FIG. 3B. Air in the combustion chamber is swirling. The first zone of combustible mixtures is formed from a centrally located injector about a spark plug at the time of sparking. The successive zones are formed continuously for the continuing injection in cooperation with the swirling air.

FIG. 3C. Same as Fig. 3B except with the injector near the periphery of the combustion chamber.

Referring to Fig. 3B: The air is caused to have a swirling motion, as indicated by the arrows. Fuel is introduced through an injection nozzle located in the center of the combustion chamber. The direction of fuel injection is constant (as compared to Fig. 3A) and it is arranged and timed so that, after the shortest possible travel, the first element of air-fuel vapor will reach the spark plug just at the desired time for ignition. The remainder of the fuel is injected continuously and, during this continuing injection, a more or less stationary flame front is formed near the spark plug, combustion products are swept away from the flame front, and fresh combustible mixture is brought to it as it is formed. For complete consumption of the air the injection is to be

terminated after a complete revolution of the swirling air. It is an extension of this scheme to use two diametrically opposed spark plugs and two fuel sprays or other multiple combinations. In this case, two or more flame fronts are established and the fuel from each spray cooperates with only its fraction of the air in the combustion chamber, the injection being terminated after the corresponding fraction of a revolution of air swirl.

Referring to Fig. 3C: The air is caused to have a swirling motion, as indicated by the arrows. Fuel is introduced through an injection nozzle located near the periphery of the combustion chamber. The direction of fuel injection is constant (as compared to Fig. 3A), and the spray shape is more or less that of a fan which cuts somewhat diagonally across the air swirl in its downstream direction. The first element of fuel injected forms an air-fuel vapor mixture which is ignited by the spark. The remainder of the fuel is injected continuously and, during this continuing injection, a more or less stationary flame front is formed near the spark plug, combustion products are swept away from the flame front, and fresh combustible mixture is brought up to it as rapidly as it is formed. For complete consumption of the air the injection is to be terminated after a complete revolution of the swirling air. In this scheme, the distance from the spark to the injector is a particular consideration since this distance will be a major factor in regulating the time during which the air-fuel vapor resides ahead of the flame front as the combustion proceeds.

EXPERIMENTAL INVESTIGATION.

For a preliminary laboratory investigation of the ideas just outlined it was desirable to make use of approximations to the idealized conditions that could be set up with more or less commercially available apparatus on single cylinder test engines. The approximation of Fig. 3B has been tested in a Wright G-200 cylinder (6.125″ bore, 6.875″ stroke) mounted on a Cooperative Universal Test Engine crankcase with a slightly modified diesel fuel-injection system sufficient to show that it is operable. The approximation of Fig. 3C has been tested in a CFR Engine [6] of 3.25″ bore × 4.5″ stroke with slightly modified diesel fuel-injection systems. For the reasons of convenience, the availability of several engine units, and better availability of more nearly satisfactory injection equipment most of the testing has been done on the CFR Engines.

Experimental Equipment-CFR Engine.

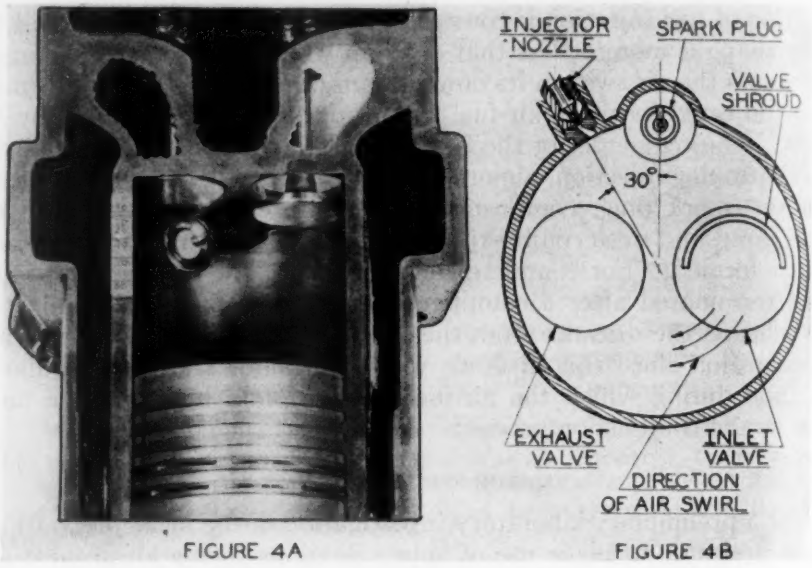
Several CFR Engine installations, of which the following is typical, have been used during the experimental investigation.

[6] This engine is used for the ASTM Standard Method of Test on Knock Characteristics of Motor Fuels D357-45.

Fig. 4A is a photograph of a sectioned cylinder showing the arrangement of this part of the apparatus. The following features are to be noted:

a. The cylinder and piston are the standard variable compression unit, substantially as used for knock testing.

b. A standard 10 mm. automotive type spark plug has been installed, with an appropriate thread size adapter, in the normal "hot plug" or "bouncing pin" hole.



Arrangement of apparatus for combustion, with control of end gas residence time, in a CFR engine cylinder.

FIG. 4A. Photograph of a sectioned cylinder showing the spark, the injector and the valve shroud.

FIG. 4B. Schematic diagram of Fig. 4A This form of diagram is used to designate future alterations.

c. The injector, which shows up as a round plug with an oval shaped hole, has been installed, with appropriate sleeves and seals to prevent water leaks, directly through the periphery of the combustion chamber wall to the left of the spark plug hole.

d. A shrouded intake valve has been installed with the shroud opening to produce air swirl directed so that the spark plug is downstream in the air swirl from the injector.

Figure 4B shows a schematic view of the cylinder arrangement. This type of diagram will be used to designate various rearrangements of these components.

The injector used in connection with Fig. 4 is a Bosch ADN o SD21, pintle type nozzle, on a standard nozzle holder. The nozzle outlet is covered with a brass deflection adapter. Fig. 5 shows a section of the adapter with the nozzle installed.

In making the cylinder openings to install the nozzles, generally it was most convenient to make the hole radially. As a result it was necessary to deflect the spray more or less in the downstream direction of the air swirl. Also, it was thought to be desirable to produce a spray shape approximating that of a fan. The deflection adapter served these purposes with this type of nozzle.

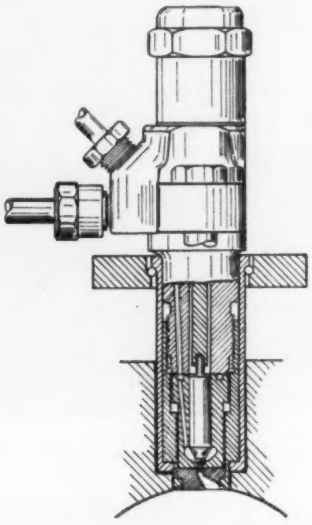


FIG. 5. Injection nozzle and detail of deflection adapter.

Figure 6 shows a close-up view of one of the test engines with the nozzle installed. Fig. 7 shows a general view of the same engine from the panel board side. This engine is connected to a General Electric Company TLC-15 electric dynamometer for power measurement and auxiliary equipment is provided for measurement and control of manifold pressure, air consumption, intake air temperature, jacket temperature, spark timing, fuel flow and other pertinent quantities.

EXPERIMENTAL RESULTS—KNOCK-FREE OPERATION.

Within certain limits, an arrangement of the various apparatus components as indicated generally by Figs. 4 and 5 results in the ability to operate the engine at practically any conditions that can be created on the test stand and, neither knock nor preignition occurs. This freedom from knock and preignition is quite independent of the Octane Number or Cetane Number of the fuel used and, within rather broad

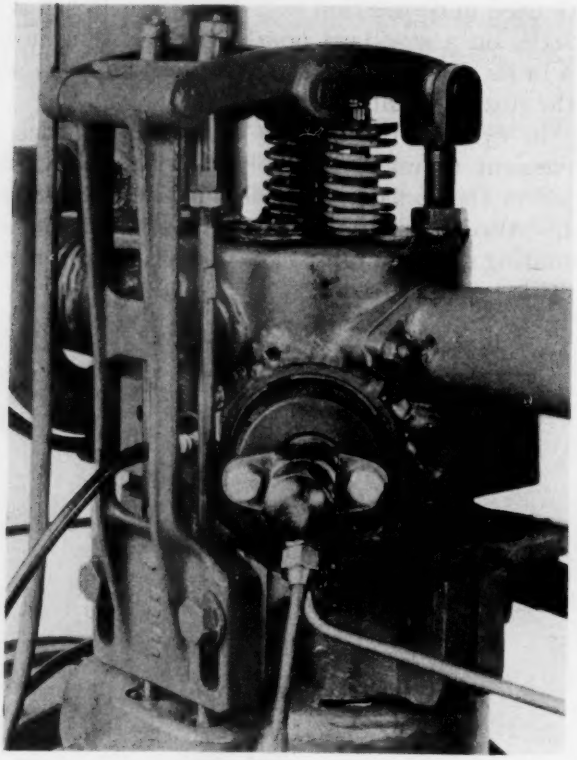


Fig. 6. Close-up view of test engine showing injection nozzle and spark plug installed.

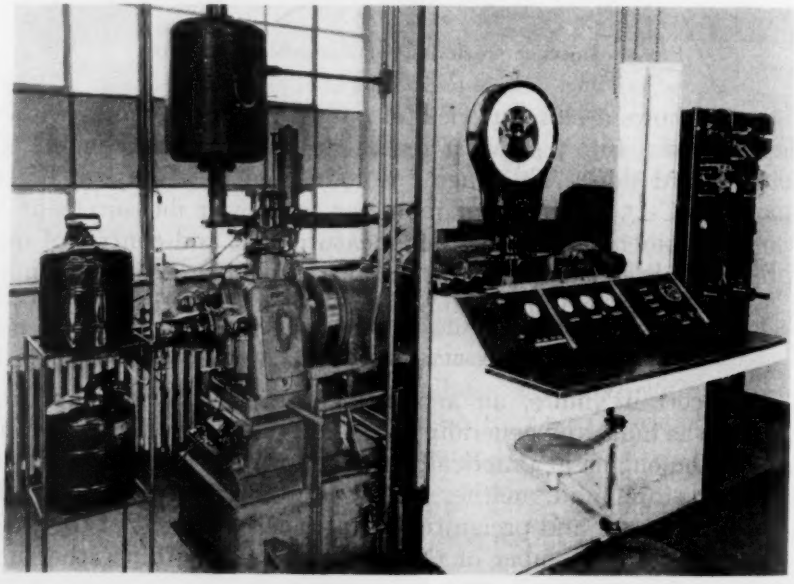


Fig. 7. General view of a typical test stand.

limits, the engine operation also is unaffected by the volatility of the fuel.

In describing the engine operation as knock-free the more or less routine check procedure has been to operate at a Compression Ratio of 10, with manifold pressures in the range of 60 to 90 inches of mercury absolute, over a wide speed range, say 300 to 3600 r.p.m., including acceleration, on two fuels, one a Straight Run Gasoline of 24 ASTM Octane Number [7] and the other, technical iso-octane having a nominal ASTM Octane Number of 100.[8] Beyond this, several specific test setups have been operated over more extreme conditions and with a wider range of fuel qualities. Some of these extremes will be described later, for the moment it is sufficient to say that an apparatus arrangement capable of meeting the routine check always has been satisfactorily knock-free under the most extreme conditions that have been imposed.

Initiating the Combustion.

To attain the conditions previously outlined, it is apparent that the initiation of combustion, knock-free operation resulting, requires that the fuel spray deliver a combustible mixture to the spark plug, that the mixture be ignited almost immediately upon arriving at the spark plug and that combustible mixture extend only over a limited portion of the combustion chamber when ignition occurs.

The spray shape, the direction of the spray and cooperative effects of the spray and the moving air combine to deliver combustible mixture to the spark plug. Experience has indicated that a wide variety of spray shapes, as determined by stroboscopic spray photographs, may be satisfactory. For a given spray shape the best direction can be selected by a simple, though occasionally tedious, cut and try procedure wherein regularity of firing is observed with systematically varied spray directions. For flexibility of engine operation it is desirable to use a nozzle, the spray from which does not change shape radically with change of fuel quantity or with operating speed.

The relative timing and location of the injector and the spark plug can be selected to cause prompt ignition of the first element of combustible mixture before additional increments of appreciable extent have formed. The range of relative timing and location within which knock-free operation occurs, independent of Octane Number, is visualized conveniently from a description of how operation depends on fuel quality for relative timings and locations outside of the knock-free

[7] ASTM Designation: D357-45 Standard Method of Test for Knock Characteristics of Motor Fuels.

[8] By converting the engine to manifold injection it was verified that, at 10 Compression Ratio and maximum power spark timing, heavy knocking occurs with iso-octane plus 6.0 ml. tetraethyl lead per gallon at manifold pressures above 35 inches of mercury.

range. A Cause and Effect summary of improper timings and locations is given by Table I.

TABLE I.

Summary of the Effects of Relative Timing and Location of Injector and Spark with High and Low Octane Number Fuels.

Cause	Effect	
	High Octane Fuel	Low Octane Fuel
1. Early Spark Timing, such that the incendiary power of the spark is gone before combustible mixture reaches the spark plug.	Engine will misfire	Spontaneous ignition and knock may occur as in a Diesel
2. Late Spark Timing; such that a substantial amount of combustible mixture has had the opportunity to form before the ignition occurs.	Engine will knock	Engine will knock*
3. Long Distance from Injector to Spark; such that a substantial amount of combustible mixture has had the opportunity to form before the ignition occurs.	Engine will knock	Engine will knock*

* The knock can occur for either of two reasons (*a*) because too much mixture exists when spark ignition occurs or (*b*) because spontaneous ignition intervenes and precedes spark ignition. The latter occurs only with very low Octane Number fuels.

Between the too early and the too late spark timings which produce a form of operation that depends on fuel Octane Number there is a zone of relative timing within which engine operation is quite independent of fuel Octane Number. This zone has been designated as the "spark tolerance" and is commonly checked on the test engines by advancing and retarding the spark timing to observe the effects listed as Items 1 and 2 of Table I. Spark tolerances commonly observed range from 10 to 20 crank angle degrees. Three conditions that affect spark tolerance are:

1. The duration of the incendiary power [9] of the spark.
2. The cycle to cycle regularity of spark timing.
3. The cycle to cycle regularity of injection timing.

In a number of cases experiments have been made where the duration of the spark discharge was varied and the earliest and latest spark timings for knock-free operations were observed. Typical results from such experiments are shown in Fig. 8.

[9] In the experimental work four ignition systems have been used more or less interchangeably without any great preference for one or another: Bosch Magneto Model ZU-1, Scintilla Magneto Model CR-4, Mallory No. 1500 Coil with the Mallory FR-10 distributor, and Delco Remy Chevrolet Coil with Mallory FR-10 single cylinder distributor.

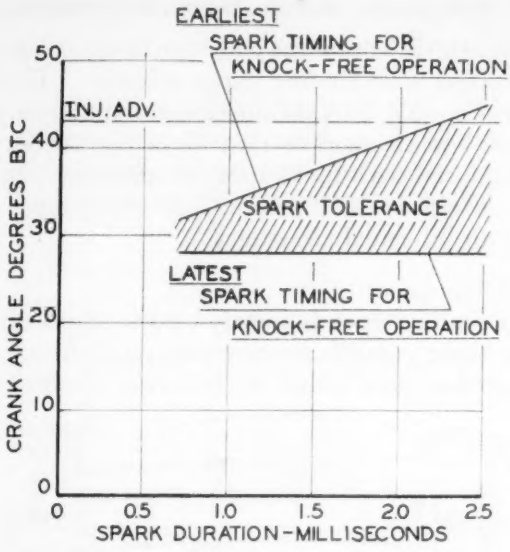


FIG. 8. Variation of "Spark Tolerance" with Duration of Spark.

Distance from the injector to the spark exhibits a maximum limit. This distance has been varied systematically by testing with spark plug positions successively more remote from the injector location as indicated schematically by Fig. 9. For the injector-spark angles of 30, 45

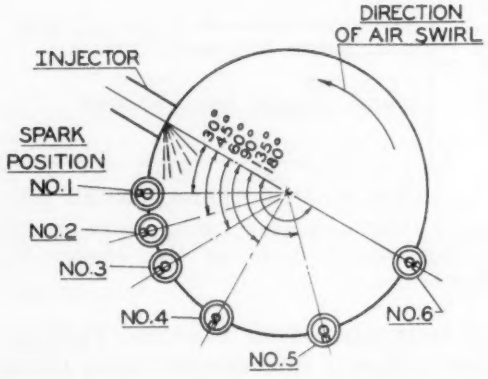


FIG. 9. One series of injector and spark plug locations used to vary the time of residence of the "leading mixture zone."

and 60 degrees, satisfactory knock-free operation could be obtained, at the 90-degree position the result was borderline and at the higher angle positions of 135 and 180 degrees the effects listed under Item 3 of Table I were observed.

EXPERIMENTAL RESULTS—ENGINE PERFORMANCE.

After ignition, the injection may be continued to produce additional increments of leading zone air-fuel vapor mixture. For full load operation the injection should be continued approximately for the time required for a complete air swirl so that all of the air may be utilized by becoming impregnated with fuel vapor, in successive increments. Part load operation is obtainable, without air throttling, by terminating the injection at an intermediate stage.

Combustion Timing.

Combustion timing depends jointly on the injection timing and the spark timing, it being possible to vary the spark timing over the "spark tolerance" range for each value of injection timing. Fig. 10 shows

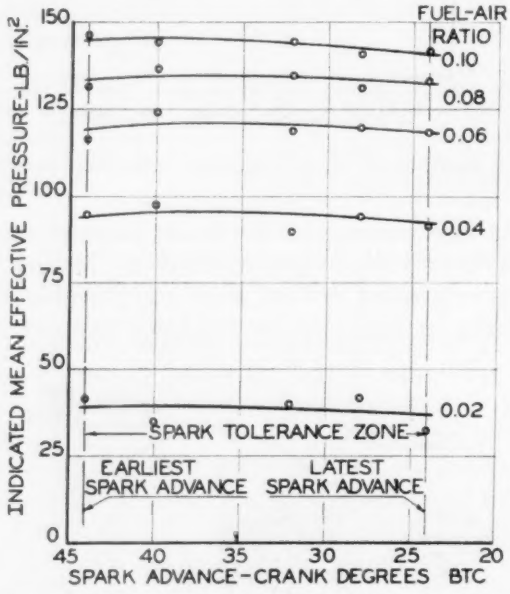


FIG. 10. Variation of IMEP with Spark Timing, within the "Spark Tolerance," for a fixed injection time. Compression Ratio 10, 45" Hg. Absolute Manifold Pressure, 1800 r.p.m., 20 ASTM Octane Fuel.

several curves of Indicated Mean Effective Pressure (IMEP) versus spark time for fixed values of injection advance, the spark time having been varied from the most advanced to the most retarded position permitted by the spark tolerance. As the variation of IMEP [10] with

[10] Because of the high friction of the CRF engine crankcase, all engine performance results are expressed as Indicated Mean Effective Pressure (IMEP) and are determined as Brake Mean Effective Pressure (BMEP) + Friction Mean Effective Pressure (FMEP). Careful planimeter measurements of a large number of indicator diagrams show the IMEP from these cards may be 5 to 10 per cent. higher than values obtained from BMEP + FMEP measurements.

spark time, within the limits of the spark tolerance, is very slight it has been general practice to set the spark at about the mid-point of the zone of spark tolerance.

Since load control is obtained by injecting for greater or lesser fractions of the duration of an air swirl, it is to be expected that the optimum time for ignition at part load should be somewhat later than at full load. Fig. 11 shows several curves of IMEP versus injection

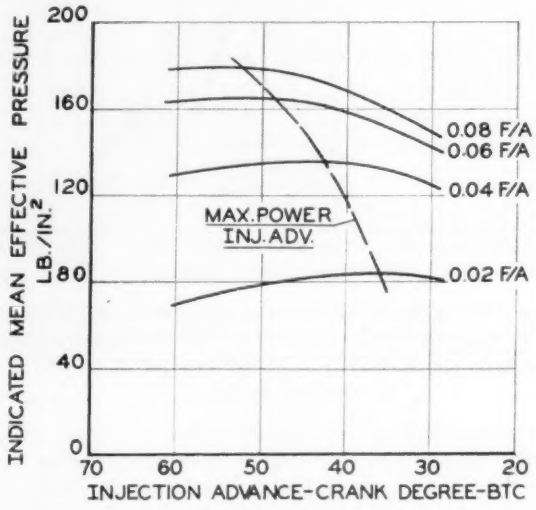


FIG. 11. Variation of IMEP with Injection Timing for different Fuel-Air Ratios. Spark timing maintained at approximate mid-point of the "spark tolerance." Compression ratio 10, 45" Hg. Absolute Manifold Pressure, 1800 r.p.m., 24 Octane Fuel.

timing which exhibit this effect; at the lower fuel-air ratios (F/A) the maximum-power injection time is somewhat nearer top center than at high fuel-air ratios. In these curves the ignition timing lagged the injection time by 8 to 12 degrees. For convenience, practically all of the testing has been at a constant injection advance of 40 to 45 degrees.

Pressure-Time Diagrams.

Pressure-time diagrams have been determined with a balanced pressure indicator for a substantial range of engine operating conditions. Fig. 12 shows a series of typical diagrams taken with the engine operating at 1800 r.p.m., with a Compression Ratio of 10, at 30 inch of mercury absolute Manifold Pressure and a 24 ASTM Octane Number gasoline and Fig. 13 shows the pressure-time diagram of Fig. 12 for 0.06 F/A replotted as a pressure-volume diagram. Two points are worthy of note; (1) that the diagrams approximate the constant volume combustion of the theotirecal Otto-cycle about as well as does normal combustion and (2) that

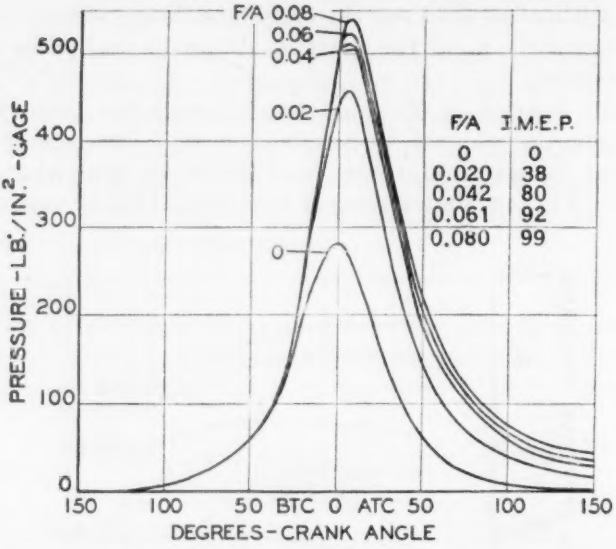


FIG. 12. Pressure-time diagrams obtained at 1800 r.p.m., 30 inches of mercury absolute manifold pressure, 10 Compression Ratio with 24 ASTM Octane Number fuel. Injection timing 45° BTC, spark timing 40° BTC.

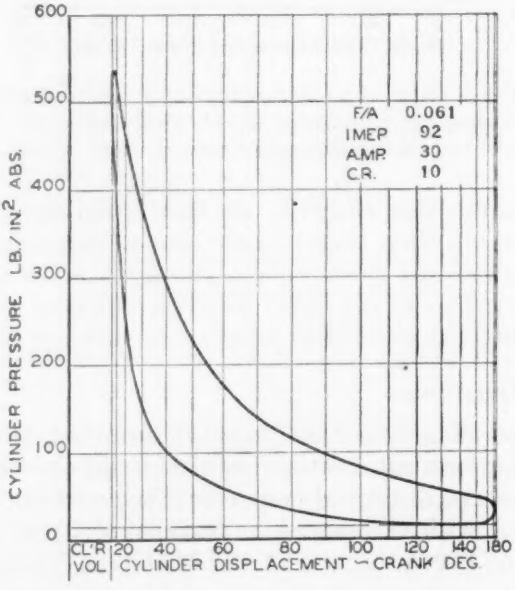


FIG. 13. Pressure-volume diagram obtained at 10 compression ratio, 30″ Hg absolute manifold pressure, 0.06 fuel-air ratio, 1800 r.p.m., 90°F. inlet air temperature, 212°F. coolant temperature. Injection begins at 45° BTC, spark at 40° BTC.

the ratio of the maximum pressure to the IMEP is well within desirable limits.

Compression Ratio.

The compression ratio has been varied from 6 through 11, higher values having been prohibited by interference of the piston with the fuel spray in the particular cylinder construction used. Fig. 14 shows

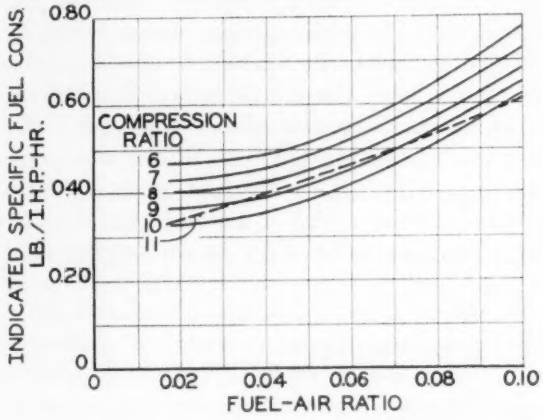


FIG. 14. Variation of ISFC (pounds of fuel per indicated horsepower hour) with Fuel-Air Ratio at Compression Ratios from 6 to 11. The curve for 11 Compression Ratio may have been affected by interference of the piston with the fuel spray. Compression Ratio 10, 45" Hg Absolute Manifold Pressure, 1800 r.p.m., 45° BTC Injection Advance, 20 Octane Fuel.

curves of Indicated Specific Fuel Consumption (ISFC),[11] in pounds of fuel per indicated horsepower hour, versus F/A for each of several compression ratios in the range from 6 to 11. Except at the compression ratio of 11 where there may have been some interference of the piston with the fuel spray, the curves show a regular decrease of ISFC with compression ratio, paralleling that which is anticipated from a theoretical efficiency analysis of the Otto-cycle.

[11] Experimental results are presented in terms of ISFC. For those who may wish to make comparisons on a thermal efficiency basis, ISFC can be converted to thermal efficiency by the equation:

$$e_t = \frac{254{,}500}{E \times \text{ISFC}}$$

where: E is the net heating value of the fuel. 19,000 BTU/lb. for the 24 Octane Fuel used in most of this testing.

e_t is the thermal efficiency in per cent.

For the 24 ASTM Octane No. fuel corresponding values are:

ISFC	e_t
0.60	22.5
0.50	26.9
0.40	33.7
0.30	45.0

Injection System Characteristics.

In conventional Otto-cycle combustion, the mixture of air and fuel-vapor in the cylinder is more or less uniform and the completeness of combustion depends upon the overall fuel-air ratio, i.e., the total weight of fuel vapor in the cylinder divided by the total weight of air. In the present form of combustion, wherein knock and preignition are eliminated, the mixture strength is to be considered in detail at successive times and places in the "leading mixture zone." In terms of the physical apparatus this includes, among other factors, correlation of the rate of fuel injection with the mass rate of air swirl and correlation of the fuel spray shape with the combustion chamber shape.

Rate of fuel injection.—The rate of fuel injection has been varied by means of an injection system designed so that it was possible to develop data for conditions in which the total fuel quantity remained constant as the period of time during which it was injected was varied. Results of two such tests are shown by Fig. 15 for engine operation at 1800

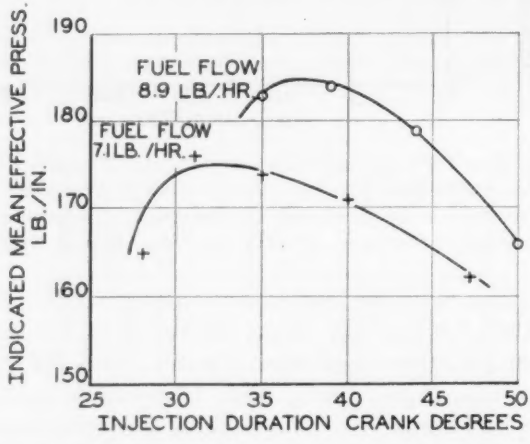


FIG. 15. Variation of IMEP with the time, measured in Crank Degrees, to inject a constant quantity of fuel. Compression Ratio 10, 45" Hg Absolute Manifold Pressure, 1800 r.p.m.

r.p.m., a Compression Ratio of 10, 45 inches of mercury absolute Manifold Pressure and on gasoline of 20 ASTM Octane Number. The total air consumption of the engine is constant for both curves, the total fuel consupmtion is a constant amount for each curve, and the measured IMEP attains a maximum value for each fuel quantity depending on the duration of the injection.

Fuel spray shape.—The fuel spray shape in the "leading mixture zone" has not been observed directly in the engine but has been subject to change through selection of fuel injection nozzle orifices to give variations in the shape of the spray and its tendency to penetrate into the air. Fig. 16 shows ISFC versus F/A curves for two different nozzles

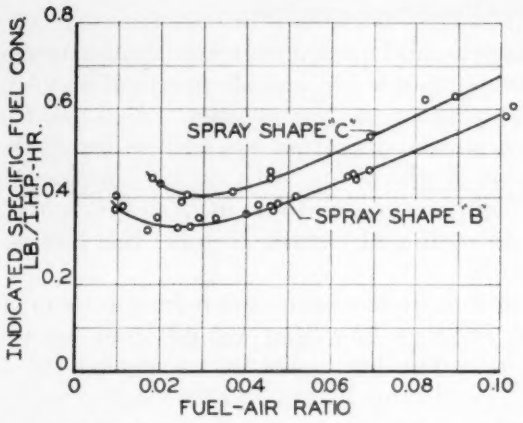
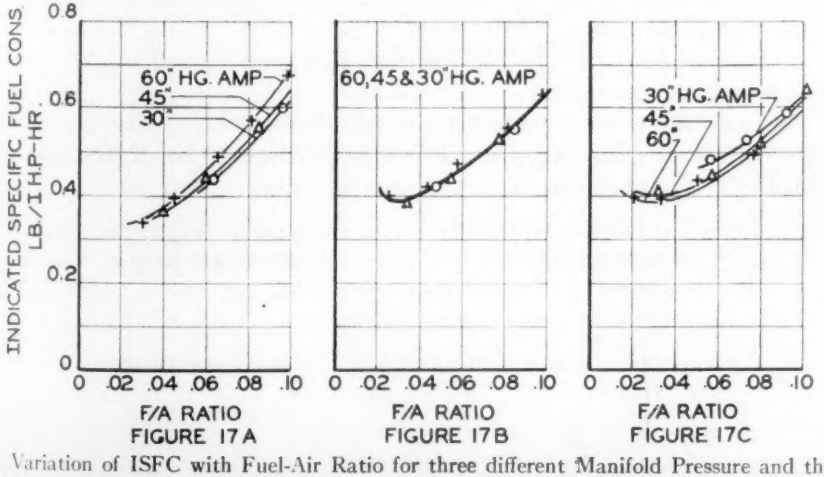


FIG. 16. Variation of ISFC with Fuel-Air Ratio for two different fuel injector nozzles.

"B" and "C." In the testing with these nozzles, other conditions of operation were comparable between the two tests. (Compression Ratio—10; r.p.m.—1800; 24 Octane Number Fuel; etc.).

Manifold pressure.—Changes in manifold pressure cause corresponding changes in the density of the air in the combustion chamber. Such density changes may alter both the mass rate of air swirl and the shape and penetration of the fuel spray. Fig. 17 shows curves of IMEP and ISFC obtained with different injection system characteristics at each of three manifold pressures. In Fig. 17A, the performance, judged in terms of ISFC, is poorer at high manifold pressure than at low manifold pressure; in Fig. 17B, the performance is practically independent of manifold pressure and in Fig. 17C it is better at the higher manifold pressures.



Variation of ISFC with Fuel-Air Ratio for three different Manifold Pressure and three different combustion arrangements. Compression ratio 10, speed 1800 r.p.m., 24 Octane Fuel.

Range of Engine Operating Conditions.

With the knock- and preignition-free combustion process, engine operation has been extended to include practically every condition that it is possible to create on the test engines. As a point of departure for these extensions, normal operation has been in the range of 1800 r.p.m. at a Compression Ratio of 10, with 90°F. Intake Air Temperature 212°F. Jacket Temperature, 30 to 60 inches of mercury absolute Manifold Pressure and with a 24 Octane Number fuel having a boiling range from 100 to 450°F.

a. Compression Ratio has been varied from 6 to 11 (see Fig. 14).

b. Manifold Pressure has been varied from 10 to 120 inches of mercury absolute. The low value of 10 seemed the lowest value of interest and the high value of 120 was the highest obtainable from the available supercharge supply.

c. Jacket Temperature has been varied from 212° to 375°F.

d. Mixture Temperature has been varied from 90° to 400°F.

e. Speed has been varied from 200 or 300 to above 3600 r.p.m. At 200–300 r.p.m. the engine will idle without air throttling, in fact, it is possible to hold the single cylinder engines at a 200 to 300 r.p.m. idle with 80 to 90 inches manifold pressure. Of course, this is not a practical operation but it is indicative of the load control obtainable from adjustment of fuel quantity only. Also, at 300 to 400 r.p.m. the engine can be loaded to full capacity and it will continue to fire regularly and develop good low speed torque.

f. Acceleration.—Running at full throttle fuel quantity the engine can be pulled down to low speed by loading the dynamometer and then, by suddenly releasing the dynamometer load, it will accelerate against the inertia forces to high speed. Alternately, idling at low speed without dynamometer load, the throttle can be opened suddenly and the engine will accelerate to high speed.

g. Indicated Mean Effective Pressure has been varied from values of 10 or 20 to over 400 p.s.i.

h. Fuels have been varied to cover almost the entire practicable range of Octane Number, Cetane Number, Preignition Resistance and Volatility. Some of the extremes that have been used are:

Fuel	Octane or Cetane Number
1. Isopentane	90 Octane Number
2. Alcohol	—
3. Benzene	—
4. Kerosene	—
5. Diesel Fuel	50 Cetane Number
6. Triptane	—
7. Iso-octane	100 Octane Number
8. n-Heptane	0 Octane Number
9. Cetane	100 Cetane Number
10. Alpha-methyl-napthalene	0 Cetane Number

When changing from one to another of these fuels there is no perceptible change in engine operation except the variation in power corresponding to the change in heating value of the fuels. Fuels 7 through 10 are of particular interest as they cover the entire defined range of Octane Number and Cetane Number. Fuel 6 has anti-knock value substantially above that available in commerical production. Fig. 18 shows curves of IMEP and ISFC versus F/A obtained on Fuels

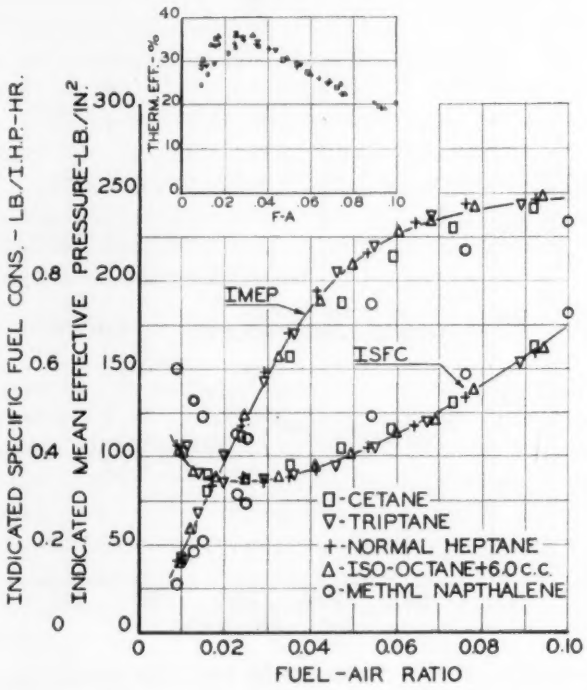


FIG. 18. Variation of IMEP and ISFC with Fuel-Air Ratio at 1800 r.p.m., 60 inches of mercury absolute manifold pressure and 10 compression ratio for five fuels which cover the defined range of Octane Number and Cetane Number.

6 through 10 at 60 inches of mercury absolute Manifold Pressure, 1800 r.p.m. and at a Compression Ratio 10 without knock or preignition occurring on any of the fuels. The insert diagram on Fig. 18 shows the results reduced to the basis of thermal efficiency. From the fact that all of the data form a reasonably good single curve, it appears that the variations of IMEP and ISFC on the main figure depend on the heating values of the fuels. These results seem adequate to demonstrate that the engine operation is quite independent of Octane and Cetane Number.

Discussion of Engine Performance Data.

In the foregoing paragraphs it was shown that the leading mixture zone, as determined by the rate at which fuel is introduced into the combustion chamber and by the shape and penetration of the fuel spray, exercises a primary influence on the efficiency of engine operation. The engine performance results previously discussed, have been obtained with conventional Diesel injection equipment used either directly as manufactured for Diesel engine use or after minor modifica-

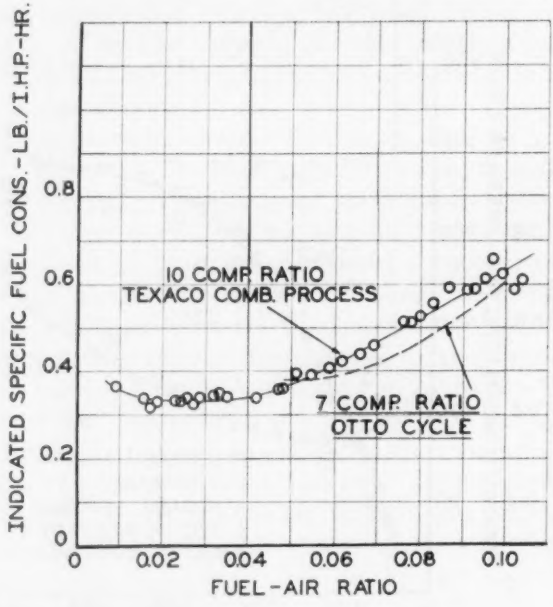


FIG. 19. Comparison of ISFC versus F/A for knock-free operation at 10 Compression Ratio with normal, full throttle, Otto-cycle combustion at 7 Compression Ratio. Engine operating conditions: Otto-Cycle, 225°F. Inlet Temperature, 212°F. Jacket Temperature, Spark Advance for maximum power, 100 + Octane Fuel, 20"–60" Hg Absolute Manifold Pressure. Texaco Combustion Process; 90°F. Inlet Temperature, 212°F. Jacket Temperature, Injection Advance 40–45° BTC, Spark Advance 35–40° BTC, 20–24 Octane Fuel, 30"–60" Hg Absolute Manifold Pressure.

tion such as the spray deflector shown by Fig. 5. Many injection system performance characteristics, desirable to obtain good thermal efficiency with the knock-free combustion process, are distinctly different from the characteristics that are desirable for Diesel engine operation and it appears reasonable to expect that some improvements in the efficiency of engine operation will result from the use of injection system components selected specially for this method of operation.

Figure 19 shows a comparison of ISFC versus F/A for the knock-free operation at 10 Compression Ratio and similar data for operation with normal Otto-cycle combustion at 7 Compression Ratio. The data for the knock-free operation were obtained with a 20 Octane Number straight run gasoline having a lower heating value of 18,900 BTU per pound and are a composite of results from several different engine operations, all of them based on conventional Diesel injection equipment, and selected as those which had given better than average results. The normal combustion data were obtained with iso-octane, having a lower heating value of 19,152 BTU per pound, with spark advance set for maximum power at each test point. Both sets of data were obtained on the $3\frac{1}{4} \times 4\frac{1}{2}$ inch CFR cylinder operated at 1800 r.p.m. and with comparable instrumentation for measurement. It would appear from Fig. 19 that the knock-free operation as carried out with Diesel injection equipment at 10 Compression Ratio, just about equals the normal Otto-cycle operation at 7 compression ratio, which is about the highest useable ratio for normal combustion with fuels of presently available Octane Number.

Summary.

From experience in the operation of Otto and Diesel cycle engines, it appears that the fuel ignition quality requirements of those engines have their origin in the residence of combustible air fuel-vapor mixture for an appreciable time at combustion chamber pressures and temperatures. This thought led to the speculation that it should be possible to carry out a combustion process, quite independent of fuel quality, by arranging the physical circumstances so that the residence time of combustible mixture at combustion chamber temperatures is made vanishingly small.

Experimental investigation shows that, when combustible mixture residence times are made quite brief, engine operation is obtained which is quite independent of Octane Number or Cetane Number and that this independence of fuel ignition quality is preserved over a very wide range of engine operating conditions.

Typical, though not necessarily optimum, characteristics of this knock-free operation are presented and the principal factors that influence the indicated thermal efficiency of the operation are described and the effects are illustrated with representative data. The conditions existing in the leading mixture zone as affected by the characteristics of the injection apparatus have particular importance. Operating with Diesel injection equipment, which has characteristics not necessarily optimum for the knock-free combustion process, the efficiency of operation at 10 Compression Ratio compares favorably with normal Otto-cycle operation at 7 Compression Ratio.

Acknowledgment.

This work was carried out as a research project of Texaco Development Corporation in the Beacon Research Laboratories of The Texas Company.

The authors wish to acknowledge the assistance of Professor C. F. Taylor of Massachusetts Institute of Technology and Professor K. J. DeJuhasz of The Pennsylvania State College, both of whom have given valuable assistance in the pursuit of this investigation. Various members of the staff of the laboratory have made substantial contributions, especially Messrs. J. R. Muenger and F. J. Villforth.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

INFRARED TRANSMISSION CURVES OF PURE HYDROCARBONS.

In connection with a program of infrared spectroradiometric research, the Radiometry Section of the Optics Division has recently installed and placed in operation a Perkin-Elmer infrared spectrometer. A feature of this program, which was inaugurated last January, is to supply transmission curves of the pure hydrocarbons being prepared by F. D. Rossini under American Petroleum Institute Project No. 44. Six such curves were recently completed and the same number are in preparation.

The equipment in use is provided with a General Motors amplifier and Brown recorder. A special feature designed and constructed in the Bureau's laboratory is a drive mechanism for continuously changing the slit width during the scanning of the spectrum to give an essentially constant energy output. The present equipment with rocksalt optics covers the range from 2 to 15 microns. A second instrument being ordered will be fitted with potassium bromide optics to extend the range to 25 microns. Progress is being made on the construction by the Instrument Shop of a research type instrument which will incorporate either one or two prisms, 11 by 15 centimeters in height and width. The apparatus is being operated by E. K. Plyler and R. Stair of the Radiometry Section.

ANALYSIS OF STANDARD SAMPLE OF CARBURETTED WATER-GAS.

The *Journal of Research* for March contains a report (RP1704) by Martin Shepherd on the analysis of a standard sample of carburetted water-gas by 24 laboratories in cooperation with Subcommittee VII of Committee D-3 of the American Society for Testing Materials. The data are presented in a series of frequency distribution plots, which show at a glance how the analyses compare with respect to each component of the gas mixture, as well as the calculated heating value and specific gravity. These plots form a clear picture of this type of gas analysis in this country. Although some very creditable work was reported, the need for some standardization is evident.

ELECTRONICS SECTION.

In order to broaden the scope of the work of the Bureau's Ordnance Development Division, an Electronics Section has been set up to handle

* Communicated by the Director.

projects involving electronics research and engineering, not necessarily related to ordnance devices.

The work of the new section includes electron tube research and development, application of electronic circuits, and radiation. These projects are at present supported largely by transferred funds from the Army Ordnance Department and Navy Bureau of Ordnance, but it is expected that new projects will accrue to the Section under the sponsorship of civilian agencies of the Government, so that it should become the Bureau's center for general development work in applied electronics.

An electronics instrumentation service is being offered by the Electronics Section. This service aims to assist the entire Bureau in the development, design, or specification of measuring and control equipment employing electron tubes. It is believed that this service by experienced personnel will prove of value in assuring full use of the advantages afforded by electronic instrumentation.

Robert D. Huntoon is in charge of the new Section. Dr. Huntoon has served in various capacities in the Ordnance Development Division since 1941, and during 1944–1945 was attached to the War Department as Expert Consultant to the Secretary of War. He performed his doctoral work at the University of Iowa, receiving his degree in 1938, after which he taught nuclear physics at New York University and engaged in research work on electron tubes at Sylvania Electric Products.

LIGHTNING HAZARDS TO NON-METALLIC AIRCRAFT.

The usual all-metal airplane constitutes an electrical shield that gives almost complete protection to its contents. Hundreds of cases are on record where such craft have been struck by lightning without serious damage. On the other hand the operation of a wood or plastic craft in a thunderstorm involves a most serious hazard. Early in the war the shortage of aluminum made it appear likely that large numbers of plywood airplanes, such as the famed British "Mosquito" bomber, would be used, and that it might be tactically necessary to tow large fleets of wooden gliders through the thunderstorm ridden air of the Southwest Pacific.

The permanent Committee on Lightning Hazards of the National Advisory Committee for Aeronautics—the government agency charged with responsibility for this field—anticipated such developments and needed answers to such questions as:

Is it impracticable to provide insulating barriers to prevent the lightning stroke from reaching and injuring the craft and its personnel and contents?

If so, can electrical conductors and metallic shields be so disposed as to lead the discharge harmlessly around personnel?

How many such lightning conductors are needed on a non-metallic airplane or glider so as to transform it from a condition in which the personnel are certain to be electrocuted to one of reasonable safety?

What material, location, and shape of conductors will give adequate protection with a minimum of weight, and a minimum of manhours in their application and maintenance?

If the lightning conductors are pared to such a minimum to save weight, that the pilot may experience a slight shock in spite of their partial protective effect, how much current can a man endure for a few millionths of a second without being incapacitated?

If a glider is towed in the vicinity of a thunderstorm how much current will flow in the tow line from the St. Elmo's Fire that develops on the aircraft under such circumstances?

Will this current be so great as to burn or melt the nylon tow line?

If so, how can such a disaster be prevented?

Is it wiser to use a nylon line and hope that it remains electrically insulating even when dragged through a wet cloud, or to use a metal line that can easily carry the St. Elmo current but which increases the likelihood that the tow will trigger off a lightning stroke?

If a metal tow line is used how large and heavy must it be if it is not to be fused by a direct lightning stroke?

Will the lightning currents weaken or melt the rudder control cable if they arc to it or flow in it?

Under the sponsorship of the Bureau of Aeronautics, Navy Department, and with the cooperation of the National Institute of Health, the staff of the High-Voltage Laboratory of the National Bureau of Standards carried out a series of laboratory experiments extending over two and a half years. In this process the large fund of information concerning natural lightning, which had been collected previously by scientists and by the engineers of electric power companies, was of great value. The experimental work involved such procedures as supporting an entire airplane from a string of insulators and measuring the current dissipated from its extremities as St. Elmo's Fire when it was charged to a potential of 1,000,000 volts; sending momentary currents up to 100,000 amperes through strips of copper and of aluminum and through bands of sprayed metal deposited on plywood; the sending of momentary currents up to 5 amperes through the forearms of volunteers from the laboratory staff and of even lethal surges of current through anesthetized guinea pigs.

The resulting series of reports that were made available to the sponsors of the project give engineering answers to the above and similar questions, and provide a firm foundation for the future possible development of lightning-proof plastic aircraft.

LITHOGRAPHIC PAPERS.

The application of a technique that was successfully used in the development of paper for war maps is involved in a basic study of lithographic paper. Paper is a thin felting of fibers normally held together by invisible bonds of a water-sensitive gel formed on the fiber surfaces by mechanical beating of the fibers in water. Substitution of insoluble bonds of synthetic resin for the gel bonds promises to lead to important advances in printing papers, particularly those for multicolor offset lithography. Objectives include the conservation of basic raw materials by permitting the increased use of widely available short-fibered pulps, and improvement in the performance of papers in printing.

By the experimental manufacture of papers in the Bureau's semi-commercial mill, information is being obtained on: (1) The extent to which papers of the usual fiber compositions can be improved as regards expansion, curling, and linting by resin bonding instead of gelatinization by beating. (2) The possibility of using a greater proportion of short resilient fibers that improve printing qualities, but do not develop suitable sheet strength by gelatinization. (3) The feasibility of making lithographic papers wholly or in part from newly-developed deciduous wood pulps for better utilization of our wood resources. Preliminary results have indicated great potential value of the resin bonding.

This investigation is a resumption of cooperative work with the Lithographic Technical Foundation that was interrupted by the war. As before, the Foundation has set up an advisory committee comprising representatives of the lithographic printing industry and of the manufacturers of paper and raw materials for paper. The advisory committee not only serves to guide the work but also is in a position to put the findings into immediate practical use.

THE FRANKLIN INSTITUTE.

STATED MONTHLY MEETING, WEDNESDAY, MARCH 20, 1946.

The stated monthly meeting of The Franklin Institute was called to order at 8:15 P.M., in the Hall of the Institute, by Mr. Charles S. Redding, President, and presiding officer.

After the National Anthem was played, the President announced that the minutes of the January meeting had appeared in full in the February JOURNAL and asked if there were any corrections.

Upon motion, the minutes were declared approved as printed.

In the absence of Dr. Henry Butler Allen, Secretary, the presiding officer called upon Dr. John Frazer, Assistant Secretary, who reported on membership as follows:

Honorary	2
Active	55
Associate	34
Student	16
Total	107

One new firm has taken out Library Sustaining Membership.

Total membership as of February 28, 1946 5,534

Dr. Frazer also reported that plans were well under way for Medal Day, which this year falls on Wednesday, April 17, 1946. He stated that reservations for the dinner were necessarily restricted by the capacity of the hall, and urged members to reply to invitations as early as possible.

There being no further business, the President introduced the speaker of the evening, Dr. W. F. G. Swann, Director, Bartol Research Foundation of The Franklin Institute, who spoke on "The Work of the Bartol Foundation During the War Years."

Dr. Swann reviewed the investigations which have been carried on by the Bartol Foundation during the period of the war, insofar as it was possible to discuss them consistently with governmental restrictions. He spoke also of present plans for investigations in cosmic radiation and nuclear physics. The talk was illustrated by slides and experiments.

The meeting was dismissed with a rising vote of thanks at 9:40 P.M.

JOHN FRAZER,
Assistant Secretary.

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknolwedged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The average cost for a print 9 × 14 inches is thirty-five cents.

The Library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from nine o'clock A.M. until five o'clock P.M., Wednesdays and Thursdays from two until ten o'clock P.M.

RECENT ADDITIONS.

AERONAUTICS.

Aircraft Year Book for 1945. Twenty-seventh Annual Edition. 1945.

ASTRONOMY.

BOK, BART J., AND PRISCILLA F. BOK. The Milky Way. Second Edition. 1945.

BIOCHEMISTRY.

RAHN, OTTO. Microbes of Merit. 1945.

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FIELD, CARTER. Bernard Baruch. 1944.

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American Society of Heating and Ventilating Engineers. Transactions 1944. Volume 50. 1945.

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BARRON, HARRY. Modern Plastics. 1945.

CAMERON, A. M. Chemistry in Relation to Fire Risk and Fire Extinction. Second Edition. 1944.

JACOBSON, C. A., Editor. Encyclopedia of Chemical Reactions. Volume 1. 1946.

SAGARIN, EDWARD. The Science and Art of Perfumery. First Edition. 1945.

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Thomas' Register of American Manufacturers. 36th Edition. 1946. Three Volumes. 1945.

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ZIMMERMAN, O. T., AND IRVIN LAVINE. Industrial Research Service's Handbook of Material Trade Names. 1946.

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BRAMSON, ROY T. Highlights in the History of American Mass Production. 1945.

MOORE, CHARLES W. Timing a Century. 1945.

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WINGATE, ISABEL B. Textile Fabrics. Revised Edition. 1945.

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MECHANICAL ENGINEERING.

DU BOIS, J. H., AND W. J. PRIBBLE. Plastics Mold Engineering. 1946.

MILITARY ENGINEERING.

JOHNSON, MELVIN M., JR., AND CHARLES T. HAVEN. Automatic Weapons of the World. 1945.

MINING AND METALLURGY.

American Institute of Mining and Metallurgical Engineers. Transactions, Volumes 145 and 160. 1941–1945.

NAVAL AND HYDRAULIC ENGINEERING.

SNOW, EDWARD ROWE. Famous New England Lighthouses. 1945.
MANNING, GEORGE C. The Basic Design of Ships. 1945.

PATENTS.

U. S. Patent office. Index of Trade-Marks 1945. 1946.

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BURTON, E. F., AND W. H. KOHL. The Electron Microscope. Second Edition. 1946.
ROUSE, HUNTER. Elementary Mechanics of Fluids. 1946.

RAILROADS.

HARLOW, ALVIN F. Steelways of New England. 1946.

SANITARY ENGINEERING.

HIRSCH, A. A. Manual for Water Plant Operators. 1945.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

Abstract of **The Effect of Ribonucleic Acid and its Hydrolytic Products and of Desoxyribonucleic Acid on Succinic Dehydrogenase and Cytochrome Oxidase.**—CHARLES ZITTLE (*Journal of Biological Chemistry*, **162**: 287, 1946). Equal weights of ribonucleic acid and hydrolyzed nucleic acid (mononucleotides) were about equally inhibitory to succinic dehydrogenase; adenylic acid and guanylic acid were both inhibitory. Inhibition was not immediate and appeared to be a slow chemical reaction rather than the intervention of an enzyme.

Desoxyribonucleic acid was more inhibitory than any of the above substances. Adenosine, which did not inhibit when tested by the manometric procedure, was inhibitory when tested by the methylene blue decolorization technique. Data obtained indicate that the inhibition was a property of the ribonucleic acid and mononucleotides and not a result of contaminating substances.

A possible important rule for nucleic acid and its hydrolytic product in relation to oxidation-reduction enzymes has been pointed out and the importance of the ratio of nucleic acid to hydrolytic products has been suggested in view of the ready diffusibility of the latter.

Ribonucleic acid and hydrolyzed nucleic acid (mononucleotides) had a negligible effect on cytochrome oxidase; the slight effect in the case of the mononucleotide preparation appeared to be due to the sodium chloride present.

BOOK REVIEWS.

MECHANICS FOR ENGINEERS, by Edward R. Maurer, Raymond J. Roark and George W. Washa. 425 pages, illustrations, 14 × 22 cms. New York: John Wiley & Sons, Inc., London: Chapman & Hall, Limited, 1945. Price $4.00.

Elaborate structures and mechanical devices of some complexity were successfully built before the science of mechanics came into being. Experience, observation, and intuitive understanding played their part. The science actually started when study of the principles of mechanics brought forth a systematization and formal statement of mechanical laws. These laws are now basic to engineering. The success of an engineer depends upon how well these laws are digested and applied—upon training.

The textbook is the outgrowth of a work published originally in 1903, entitled Technical Mechanics, and subsequently rewritten a number of times. The present writing reflects experience in teaching the subject and makes the coverage adequate and impressive. It is divided into statics and dynamics, the former starting with a discussion on force and force systems and proceeding through treatments on composition and resolution of forces and forces in equilibrium. Simple frameworks of the truss and crane type preceed discussions on friction and center of gravity and centroid. A brief reference to suspend cables completes the treatment on the static section of the work.

The larger portion of the book is devoted to dynamics and it is quite rigorous Beginning with rectilinear and curvilinear motion the progression is through rotation and plane motion of a rigid body, D'Alembert's Principle, work and power and energy, momentum and impulse, through dimensional motion of a rigid body, systems and dimensions of units, moment of inertia and radius of gyration, mechanical vibrations.

This text is thorough. Much weight is placed on the use of solving problems as illustrations to make the principles useful. These are numerous and coupled with directions in methods of proceeding. Considerable space is devoted in the back of the book to a large number of carefully graded problems with proper reference to the part of the text which is applicable. Complete or partial answers are given to many of these problems. The book does not offer an easy road to mastery but it does guide the serious minded and industrious student to a valuable knowledge of the subject.

R. H. OPPERMANN.

INSIDE THE VACUUM TUBE, by John F. Rider. 407 pages, drawings, tables and illustrations, 15 × 22 cms. New York, John F. Rider Publisher, Inc., 1945. Price $4.50.

The vacuum tube industry and industries using vacuum tubes require the services of a great many people, and the outlook is for even a greater number. Many are unfamiliar with the construction and operation of the tubes, many do not have sufficient educational background to become familiar through the usual text. The purpose of the book at hand is "to present a solid, elementary concept of the theory and operation of the basic types of vacuum tubes as a foundation upon which can be built a more advanced knowledge of tubes in general."

In the beginning certain facts are given relating to the operation of the vacuum tube: the manner in which and the reasons why the electrons are liberated; the basic reasons for the movement of electrons between charged surfaces, or the underlying basis for plate current within the tube. With this as a foundation, investigation is made as to what happens in the different types of vacuum tubes, and the variation in the magnitude of the inter-electrode electron stream as the result of changes in the strength of the electrostatic fields which exist in the vacuum tube. After reference to the diode, the triode is taken up giving both static and dynamic characteristics Special attention is directed to the graphical illustration of the process of amplification and the various subjects which pertain to it Explanations are given

tetrodes and pentodes and then consideration is made of the action of the cathode circuit in the vacuum tube, which leads to a treatment of power amplifiers. The last chapter is devoted to miscellaneous vacuum tubes.

The book is elementary. The electron theory in the very beginning is of such simple and plain language that anyone regardless of previous education can understand it. The sequence of method of presentation as well as topics is toward the more complex, but emphasis is always on the elementary. An innovation in the book is the use of the line anaglyphs that provide three dimensional pictures. Many employees in the radio and electron tube industries can benefit by the use of the book.

R. H. OPPERMANN.

PUBLICATIONS RECEIVED.

Atomic and Free Radical Reactions, by E. W. R. Steacie. American Chemical Society Monograph Series. 548 pages, illustrations, 15 × 23 cms. New York, Reinhold Publishing Corporation, 1946. Price $8.00.

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CURRENT TOPICS.

ARMY AND NAVY NOTES.

Synthetic Rubber Latex.—Wartime research and rapid progress in the use of synthetic rubber latex are speeding the return of latex foam products to civilian markets, the Rubber Manufacturers Association said recently.

Natural rubber latex was the basic material for the revolutionary new cushioning products introduced in the pre-war decade. Supplies were choked off when the Japs swept into Malaya and the Netherlands Indies in 1941. Resumption of natural rubber latex shipments from the Far East is not expected in the immediate future.

Meantime, the past year has seen manufacturers making rapid progress in adapting synthetic latex to new uses. Improvements in recent months have been such that latex foam made from synthetic rubber can step in to meet most of the backed-up civilian demands as soon as government restrictions are lifted.

Latex foam was developed in 1931. Hailed as a revolutionary new cushioning and mattress material, it had become popular and had been convincingly tested and used by the public before the war.

It went to war on Pearl Harbor Day. From that day to this, both natural and synthetic latex foam products have seen continuous war service, on land, on sea and in the air.

As crash padding, it saved hundreds of General Patton's rough-riding tank corpsmen from serious injury as they bounced around in Shermans and Pershings. Army medics have used it for hospital mattresses and stretcher pads and to equalize pressure under cumbersome casts and splints.

In the air, latex foam is serving as shock absorbers for delicate flight instruments, on oxygen masks and earphone pads, and to seal bomb bay doors on such aerial giants as the B-29, where it is also employed as cushioning material to combat crew fatigue on long hauls to Tokio. The Air Corps also uses it for parachute seats and backs and gunners' pads.

At sea and in landing operations, it seals amphibious tanks, serves as submarine mattresses and pillows, pads deck helmets and soaks up battle shock on gun control apparatus and mines.

The industry promises latex foam a brilliant future. An odorless, breathable matrix of millions of tiny air cells and soft resilient rubber, its extraordinary cushioning properties and its long life will return it rapidly to its prewar place in the home, public buildings and office buildings and in public and private transportation.

Latex foam made its transportation debut in 1935 when it went into the first streamlined train. It has been specified for mattresses and cushioning in every Pullman built since that time. By 1938 it had spread rapidly to intercity buses, commercial airliners and to steamships, and in the same year it was introduced to civilians in the de luxe mattress line.

It gained quick acceptance in the auto industry. Just before the war it was offered as standard equipment cushioning for a number of top lines in the upper price bracket and as optional equipment in intermediate lines. The automobile manufacturers are today planning its use in the car of tomorrow.

Widespread use in the field of upholstered furniture and mattresses is forecast by manufacturers, the association said.

While use in mattresses, public and private transportation, hospitals and public buildings, and as cushioning materials in the furniture industry will represent the greatest proportion of the market, latex foam had found many other applications before the war.

Latex foam has been used in plastic surgery with singular success, and in molding soft, pliant, life-like forms for use by patients who have undergone operations for cancer of the breast. The material also had found popular use in the foundation garment industry where it was particularly adapted in forming brassieres. An extensive post-war market is seen in this field.

In that it can be easily washed and is highly resistant to germs, moths and vermin—factors which have made it attractive to hospitals for many uses—latex foam has also been employed widely in the manufacture of multi-colored soft toys for children, bath mats, shaving brushes, powder puffs, soap dishes, mop heads and like items.

R. H. O.

Giant Loud Speakers.—Powerful voices from two miles up in the sky blasted the news of unconditional surrender to isolated Japs still holding out in jungles, caves and swamps on remote islands of Hirohito's once far-flung empire. New airborne radio loudspeaker equipment, operating at a power more than 1000 times that of an ordinary loudspeaker, was rushed to the Pacific early in September to aid in the huge task of disarming all Jap fighting men and in directing civilian movements.

On May 14, the Navy ordered the new loudspeaking equipment, known as the Navy "Polly Project," and requested delivery within 100 days. Bell Telephone Laboratories completed the design and Western Electric manufactured the equipment, ready for shipment, just 77 days later. The giant loudspeakers, which were installed in three four-engined Privateer bombers, are capable of making the human voice audible over an entire city from a height of 10,000 feet. One of the planes, just before taking off for the Pacific, flew over the Western Electric plant at Kearny, N. J., and broadcast a message in recognition of the fast production job done by the Kearny Workers.

The Navy order was a direct tribute to the older "Polly" equipment and the gallant crew who used it successfully in battering down the Jap will to resist on Wotje, Saipan, Iwo Jima and Okinawa during the last stages of the war. "Polly" was first directed at Jap ears on Wotje Atoll after months of meticulous training at Cherry Point, N. C., and Oahu Island in Hawaii. Flying at 2700 feet, within machine gun range, a tough Ventura PV 1 twin-engined plane, carrying a crew of six, slowly circled the atoll and then blasted startled Jap ears: "Attention, Japanese soldiers of Wotje Atoll, attention!" A short news broadcast followed, then a selection of Japanese popular music, a short propaganda talk and finally more news. The whole program took about fifteen minutes and was given twice.

"The first time we flew over," said Lieutenant Melvin C. Snyder, of Denver, Colorado, a technical observer who handled the "Polly" controls, "we couldn't see a Jap through our binoculars. It was a terrific let-down—like playing to an empty house—although we knew there were 5,000 Japs

isolated on that by-passed island of only five square miles. On the next day we saw them and it was the *only* time we saw them. They gathered in little groups and waved their arms wildly. On the third day we flew over they shot at us with everything they had. They knocked a chunk of our plane loose, and after that we never flew below 5000 feet."

Every day for two weeks the voice from the sky worked on the Japs. They were told to be on the alert for an American landing craft which would land some day soon and pick them up. When the LCI beached on the atoll only nine ragged Japs, who had succeeded in evading their own troops, appeared for surrender. In succeeding campaigns, however, the bag of prisoners increased rapidly. "We didn't even count the Koreans," said Lieutenant Snyder.

The sky programs were prepared by a language officer, Lieutenant Daniel Stempel of Brooklyn, a former instructor of English Literature at Harvard University. They emphasized straight news broadcasts to the isolated Japs completely ignorant of the war's progress. Promises of good American food, medical aid and fair treatment helped coax the Nips into surrendering, many of whom were starving and ridden with disease. The broadcasts also hammered away at the futility of suicide or further fighting. "Killing yourselves won't help Japan's future", bellowed the airborne bullhorns. Usually a Nisei boy transcribed the scrip onto records. On very urgent occasions he accompanied the crew and broadcast directly.

In succeeding campaigns the flak grew thicker every day, a testimony to the threat of "Polly" to Jap morale. On Okinawa, where the Japs had concentrated their most effective artillery, the Ventura was hit several times but it miraculously escaped being brought down. Finally in July the tough Ventura became so badly battered the crew abandoned it after removing the equipment.

"The Western Electric equipment was even tougher than the plane," said Snyder. "It's amazing how it withstood terrific heat, coral, salt water and high pressure wear and tear without once going into a blink. Not even one tube had to be replaced."

R. H. O.

Radar in Navigation.—Radar which has played and is still playing a highly important role in the war was exhibited in its postwar form for the first time recently when General Electric demonstrated aboard a maritime training ship an "electronic navigator" that can detect through darkness, fog and storm the position of any above-water obstacles, such as lighthouses, buoys, icebergs, other ships and land, at distances up to 30 miles, depending upon the size and shape of the object.

This device, to be manufactured for civilian use as soon as material restrictions are lifted, operates on the radar principle of radio waves which are reflected from objects and are measured to give true bearing and distance of the object from the point of sending.

Dr. W. R. G. Baker, General Electric vice president in charge of electronics and an engineer who had much to do with the development of television in its present form, predicted that the new device will revolutionize "thick weather" navigation, providing the mariner with an instrument to plot a safe course, even though his normal visibility is strongly limited by natural conditions.

Held aboard the U. S. Maritime service training ship, "American Mariner," the working demonstration showed, in the dead of night, how it is possible for a ship to operate safely and accurately through any kind of thickened overcast condition, with the radar mechanism giving the ship's navigator a bird's-eye view of the surrounding waters, his own ship being always in the center of the field.

During the demonstration in the waters off Long Island, such objects as other ships, channel markers, lighthouses and land masses were shown in their relative positions, and both distance and bearing were automatically plotted on the face of the viewing screen in relation to the "American Mariner's" own course. The distance of objects from the ship was shown in true proportion, being measured by a series of concentric "marker rings" electronically super-imposed on the picture screen. According to General Electric engineers, the measurement of distance so given is accurate to the point of 1 per cent.

Basis of the "electronic naviagtor," it was explained, is a rotating antenna, located on the top deck of the ship and analogous to a searchlight, in that it sends out beams to locate obstacles in the ship's path. The difference, however, is that beams from the radar antenna, which are actually powerful radio micro-waves, are capable of penetrating fog or any other atmospheric conditions without hindrance. Moreover, they are sent out as "pulses" or surges of extremely short duration and at a very rapid rate.

As the radar waves locate an obstacle in the surrounding waters, they bounce off and are scattered, no matter what material the object is. Some of these echoes—or scattered waves—will return to the rotating antenna, which also acts as the receiving antenna during the time intervals between the outgoing pulses. After being amplified, these echoes are made to appear as bright spots on the face of a cathode ray tube, which is somewhat similar to a television screen tube. The image thus formed gives the operator a "radar picture" of the obstacle, and the marker rings tell him how far away it is.

By controlling internal circuits, the operator may change the scale of the field to cover either a 2,6, or 30-mile radius. Thus, when a ship is sailing in the open sea, the operator will use the 30-mile range until an object approaches to within 6 miles. Then, by turning a knob on his radar set, he is immediately presented with a larger scale chart, the outer radius of which is 6 miles. For very close work another turn of the knob provides a 2-mile radius chart on which objects may be observed down to about 200 yards.

R. H. O.

Fracture Cast Knitted of Glass and Plastic.—Promise of greater comfort and freedom of movement for those suffering from fractures or other injuries that require immobilization of the part affected, is provided by a newly developed, flexible, Fiberglas and plastic bandage. The bandage is immersed in a setting solution just before it is applied to the injured part, causing the bandage to harden into a rigid cast. Dr. Roger Anderson and Herbert R. Erickson, of Seattle, describe the bandage and the method of applying it in an article entitled "Glass Plastic Cast" in the (September 1945) issue of the American Journal of Surgery.

Surgeons have long recognized that the plaster-of-Paris cast, invented

nearly 100 years ago (1852), possesses certain disadvantages, notably heavy bulk and weight and poor penetrability of X-rays. The Fiberglas-plastic cast weighs only one-fourth to one-sixth as much as an equivalent plaster cast and does not block X-ray penetration. Its clinical effectiveness has been established by application to a substantial number of patients at a United States Naval Hospital.

The principles upon which the new cast is based were conceived some years ago by Dr. Anderson. In its original form the bandage was made by loosely knitting a yarn composed of a mixture of cellulose acetate and a regenerated cellulose rayon. The setting liquid was sprayed on after the bandage had been applied. The resulting cast had the desired properties of light weight, small bulk and penetrability by X-rays, but care had to be taken to avoid applying the bandage under tension because this sometimes led to contraction of the cast; also the spraying on of the setting liquid required considerable time.

These drawbacks were eliminated by replacing the regenerated cellulose rayon content with Fiberglas yarn which imparts permanence of dimension to the cast as the setting liquid dries. The preferred composition is cellulose acetate 80 per cent. and Fiberglas 20 per cent., by weight. The knitted bandage is soft, flexible, and elastic. It can be applied under tension without danger of subsequent contraction. Because it can be immersed in the setting liquid before it is applied, much time is saved.

The resulting cast has a silky interior finish. The open mesh provides free circulation of light and air, making the cast cool to wear. Since the components do not absorb water, the patient can bathe while wearing it. Fiberglas is inert as far as body tissues are concerned, and the other components have been chosen for their nontoxic and non-irritant characteristics. The cast can safely be placed over open wounds. No allergic reactions to it have been reported.

R. H. O.

Battle Performance of Radar Demonstrated.—Invisible waves, rebounding from a shiny metal plate and illuminating a fluorescent tube twenty feet away, in a recent demonstration duplicated battle performance of radar—"sixth-sense" weapon that seeks out distant and unseen enemy targets and reports their whereabouts with split-second speed.

Dr. J. A. Hutcheson, Associate Director of the Westinghouse Research Laboratories, showed how radio waves, beamed across hundreds of miles and through darkness, clouds and fog, bounce back from the target and are picked up at the source of origin by a sensitive receiver that records the exact location and distance of an enemy plane, ship or land installation.

"One of the most valuable weapons developed for this war," Dr. Hutcheson said, "radar was used to detect enemy planes, ships and fortifications; to direct the fire of big guns; to guide friendly planes; to aim searchlights with unerring precision; and to steer Allied ships through dangerous or fog-ridden waters."

To show how radar works in actual service, Dr. Hutcheson first removed an ordinary fluorescent tube from an overhead socket and held it a few feet to

the side of a special laboratory radar set. At the opposite end of the room, some twenty feet away, an assistant held a shiny metal plate facing the tube.

Then the Westinghouse scientist threw a switch, sending an invisible beam of radio waves from the radar transmitter streaming toward the plate. The beam rebounded from the shiny surface and was reflected directly back to the tube in Dr. Hutcheson's hand. Its invisible rays excited the phosphor coating of the tube, illuminating it as brightly as though an electric current had passed through.

Although radar has reached a high stage of development only in recent years, Dr. Hutcheson pointed out, Westinghouse research engineers had done important pioneer work as early as 1932. One research scientist, Dr. Ilia E. Mouromtseff, experimented with ultra-high frequency radio transmission between the roof of a Westinghouse plant at East Pittsburgh and the Research Laboratories a mile away.

It was during these experiments that Dr. Mouromtseff noted that radio waves were reflected from the metal surfaces of the automobiles that passed along the Lincoln Highway lying in the path of the beam.

Dr. Mouromtseff also designed and developed the first electronic tube for use in radar sets for the United States Signal Corps. The tube was produced in quantity and was present in the sets that detected Japanese planes thirty-two minutes before they reached Pearl Harbor on December 7, 1941.

Postwar Uses Outlined.

Most wartime applications of radar, Dr. Hutcheson declared, are readily convertible to peacetime uses.

"The use of radar to detect enemy planes," he said, "can be put to work in traffic control at airports, where there should be a tremendous increase in the number of daily take-offs and landings. A radar set in the control tower will give the operator a complete picture not only of all planes approaching the field but also of their position in relation to landing strips, obstructions and other planes. This information, radioed to incoming pilots, will enable the operator to control large numbers of planes with great speed and accuracy."

The use of radar on commercial and private planes will undoubtedly contribute to the reduction of air accidents, the scientist asserted.

"The possibility of aircraft crashing against mountainsides or other lofty obstructions will be greatly lessened," he said, "because radar penetrates the deepest fog, the most blinding snowstorm. Complete information concerning the obstruction—its height and distance from the plane—would be flashed to the eye of the pilot in ample time to avoid a collision."

Similarly, radar will prove a peacetime boon in guiding ships through dangerous waters by warning them of obstructions hidden by fog or darkness, Dr. Hutcheson said.

R. H. O.